Mathematics 181 **May** 7, 2012 **Final Exam**

____
Name

Technology used: ______________________ **Only write on one side of each page.**

1. [15 points] Do **one** (1) of the following.

   (a) Using Riemann sums, carefully explain why the formula for the method of slicing, $\int_a^b A(x)\ dx$ gives the volume of a solid. Include the meaning of a, b, and $A(x)$.

   (b) If $\{a_n\}$ is an infinite sequence of numbers, **fully describe the definition** of what it means to say the infinite series $\sum_{n=1}^{\infty} a_n$ converges to the number L.

2. [15 points] Find the exact length of the curve given by $x = \frac{1}{6}y^3 + \frac{1}{2}y^{-1}$ from $y = 2$ to $y = 3$.

3. [15 points] Solve the following initial value problem. Express your answer y as a function of x.

$$\sec(x)\frac{dy}{dx} = e^{y+\sin(x)}, \quad y(0) = 0$$

4. [15 points] Determine the exact sum of the convergent geometric series

$$\sum_{n=2}^{\infty} (-1)^n \frac{3^{n-1}}{5^n}$$

5. [15 points] Find the radius and interval of convergence of the following power series. Also determine any values of x for which the series converges conditionally.

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{n+2}}{2n+1}$$

   w

6. [15 points] Determine the Taylor Series for the function $f(x) = (x+3)^{-2}$ when $a = -1$.

7. [15 points each] By hand (without using a calculator or table of integrals), evaluate **two** (2) of the following integrals

   (a) $\int \frac{[\ln(t+1)]^2}{t+1} dt$

   (b) $\int x^2 e^{4x}\ dx$

   (c) $\int \sqrt{1-9t^2}\ dt$

   (d) $\int \frac{4x^2}{(x-1)(x^2+2x+1)} dx$

8. [15 points each] Do **two** (2) of the following:

   (a) Find the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{2 \cdot 5 \cdot 8 \cdot \cdots \cdot (3n-1)}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)} x^n$$

   (b) Prove that if all of the terms a_n are positive and the series $\sum_{n=1}^{\infty} a_n$ converges, then the series $\sum_{n=1}^{\infty} a_n^2$ also must converge.

   (c) Prove the theorem that absolute convergence implies convergence. More specifically, **prove** that if the series $\sum_{n=1}^{\infty} |a_n|$ converges then so does the series $\sum_{n=1}^{\infty} a_n$.

Useful Information

Taylor's Formula

If f has derivatives of all orders in an open interval I containing the number a, then for each positive integer n and for each x in I, we have $f(x) = P_n(x) + R_n(x)$ where $P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)(x-a)^k}{k!}$ and $R_n(x) = \frac{f^{(n+1)}(c)(x-a)^{n+1}}{(n+1)!}$ for some number c between a and x.

Frequently Used Taylor Series

- $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, for $|x| < 1$
- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, for $|x| < \infty$
- $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$, for $|x| < \infty$
- $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$, for $|x| < \infty$
- $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$, for $-1 < x \leq 1$