Mathematics 181A **Fall 2010**

Exam 4

November 18, 2010

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Name

Technology used: ____ **Only write on one side of each page.**

Show all of your work. Calculators may be used for numerical calculations and answer checking only.

Do any two (2) of the following.

A. (15 points each)

Use appropriate tests (Divergence, Direct Comparison, Limit Comparison, Ratio, Root, or Integral) to determine if the following series converge or diverge. Give reasons for your answers.

(a) $\sum_{n=1}^{\infty} \frac{2n^2+6n}{\sqrt[3]{5n^8+6}}$

(b) $\sum_{n=1}^{\infty} \frac{n^{99}}{3^n}$

(c) $\sum_{n=1}^{\infty} \frac{(n+2)!}{n!3^n}$

Do any four (4) of the following problems

1. (5, 5, 5 points) Do all three of the following.

   (a) Write out the first seven terms of the sequence that is given by the recursion $a_1 = 2$, $a_2 = -1$, and for $n \geq 1$, $a_{n+2} = \frac{a_{n+1}}{a_n}$

   (b) Find a formula for the nth term of the sequence that starts $1, 5, 9, 13, 17, 21, \cdots$

   (c) Find a formula for the nth term of the sequence that starts $1, 0, 1, 0, 1, 0, 1, 0, \cdots$

2. (8, 7 points) Determine whether or not the following seqences converge or diverge. If they converge, find the limit.

   (a)

   $$a_n = \sqrt[n]{3^{2n+1}}$$

   (b)

   $$a_n = n - \sqrt{n^2 - n}$$

3. (15 points) The following is a convergent telescoping series. Find the sum. Show all of your work.

$$\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+2}} \right)$$

4. $(8, 7$ points) The following is a geometric series involving the variable x.

$$\sum_{n=1}^{\infty} 2\left(\frac{x-3}{2}\right)^n$$

(a) Find all values of x for which the series converges

(b) Give the sum of the series when it converges.

5. (3 points each) For each of the following infinite series write down a p-series or a geometric series for a direct or limit comparison test that would determine if the given series converges or diverges. DO NOT MAKE THE COMPARISON. Just write down the series that you would compare to.

(a) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

(b) $\sum_{n=1}^{\infty} \frac{1}{3^{n-1}+1}$

(c) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+3}}$

(d) $\sum_{n=1}^{\infty} \frac{\sqrt[n]{n}}{n^2}$

(e) $\sum_{n=1}^{\infty} \frac{(\ln n)^4}{n^3}$

Do ONE (1) of the following problems

B. (10 points) Let

$$a_n = \begin{cases} \frac{n}{2^n}, & \text{if } n \text{ is a prime number} \\ \frac{1}{2^n}, & \text{otherwise.} \end{cases}$$

Does the series $\sum a_n$ converge? Give reasons for your answer.

C. (10 points) Prove that if $\sum a_n$ is a convergent series of nonnegative terms, then $\sum (a_n)^2$ also converges.

D. (10 points) **Use the definition** of the limit of a sequence given below to prove that the sequence $\{a_n\}$ where $a_n = \frac{2n+3}{n+1}$ converges to 2.

Definition: To say $\lim_{n\to\infty} a_n = L$ means: given any positive number ε there is a number N such that whenever $n > N$ then $|a_n - L| < \varepsilon$.