Mathematics 181A — Fall 2010

Exam 3

October 27, 2010

____
Name

Technology used: ______________________________ **Only write on one side of each page.**
Show all of your work. Calculators may be used for numerical calculations and answer checking only.

You must do this problem.

A (4, 11 points) Given the rational function below.

(a) Verify it is a proper fraction and that the denominator is a product of linear and irreducible quadratic factors.

(b) Write out the partial fraction decomposition. **Do not solve** for the constants.

$$\frac{3x^9 - 7x^8 + 4x^5 + 4x^2 - 34x + 2010}{x^3(x+7)^2(x^2+2x+5)^3}$$

Do any five (5) of the following problems

1. (15 points) Without using a table or a calculator, evaluate the following integral

$$\int \frac{6x^2 + x + 8}{x^3 + 4x}\,dx$$

2. (8, 7 points) Which formulas in the table of integrals passed out with this examination are appropriate for solving these two integrals? You will need to make substitutions first. DO NOT EVALUTE THE INTEGRALS

(a)

$$\int \frac{e^{2x}dx}{1 + e^{4x}}$$

(b)

$$\int \frac{x+7}{\sqrt{x^2 + 4x - 5}}\,dx$$

3. (15 points) A function f and its derivatives satisfies the following for all x in $a \le x \le b$: $f(x) < 0$, $f\,'(x) > 0$, $f\,''(x) > 0$.

Which of the numerical estimates left endpoint, right endpoint, midpoint, or trapezoid rules) for $\int_a^b f(x)\,dx$ will always produce an overestimate? Which will always produce an underestimate? And for which is there not enough information to determine the relationship of the estimate to $\int_a^b f(x)\,dx$?

4. (5, 10 points) Given the function $f(x) = e^{-x^2}$.

(a) Show the work justifying $f''(x) = (-2+4x^2)\, e^{-x^2}$.

(b) Estimate the smallest value of the integer n necessary to guarantee that the Trapezoid rule will approximate the integral

$$\int_{-1}^{1} e^{-x^2}\, dx$$

with an error less that 10^{-8}.[Useful Information: the error bound formula for the Trapezoid Rule is $|E_n| \leq \frac{M}{12}\frac{(b-a)^3}{n^2}$.]

5. (15 points) How many integrals with a single impropriety at a limit of integration are required to evaluate $\int_{-\infty}^{\infty} \frac{x}{(x-1)(x+4)} dx$? Give reasons for your answer.

6. (15 points) Without using a table or a calculator, evaluate the integral

$$\int \frac{1}{y\left([\ln(y)]^2 + [\ln(y)] - 2\right)}\, dy$$

7. (15 points) Do **one** (1) of the following. You may use the table of integrals for this problem.

(a) Determine if the following integral represents a number. If it does, find it. If it does not, explain why.

$$\int_{-2}^{3} \frac{1}{(x+1)^2}\, dx$$

(b) Show that the following improper integral diverges either by directly computing the limit or by comparing to an appropriate integral.

$$\int_{2}^{\infty} \frac{x\, dx}{\sqrt{x^4-1}}$$