Mathematics 181A **Fall 2010**

Exam 1

September 16, 2010

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Name

Technology used: ____ **Only write on one side of each page.**

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

1. [10 points] Rewrite the following sum as indicated.

$$\sum_{k=4}^{101} \left(k^3 - 1\right)^2 = \sum_{j=9}$$

2. [15 points] Find the derivative of $G(x) = \int_{e^{5x}}^{3} \sin\left(t^2\right)\, dt$ using part 1 of the Fundamental Theorem of Calculus.

3. [15 points] Do **one** (1) of the following. Do not use your calculator.

1. (a) Evaluate $\int \left(\frac{2t^2+1}{t^3} - 3t^{\sqrt{2}} + 5\sec^2(t) + 6\sec(t)\tan(t) + \frac{4}{1+t^2}\right)\, dt$
   (b) Verify the formula $\int \arcsin(ax)\, dx = x\arcsin(ax) + \frac{1}{a}\sqrt{1-a^2x^2} + C$ where a is a constant by differentiating the right hand side.

4. [5, 5, 10 points] If we use the partition points $x_0 < x_1 < x_2 < \cdots < x_n$ to partition the interval $[2, 5]$ into n subintervals of equal length

1. (a) What is the value of Δx in terms of the letter n?
   (b) Write the values of x_0, x_1, x_2, x_k, and x_n in terms of the letter n.
   (c) Use sigma notation to write, in terms of the letter n, the Riemann sum for the function $f(x) = x + x^2$ that uses the **left** endpoint of each subinterval as the value of c_k.**Do not simplify this Riemann Sum.**

5. [10 points each] Do **both** of the following. Do not use your calculator. [Useful information: $\cos(\pi/3) = 1/2$ and $\cos(\pi/4) = 1/\sqrt{2}$.]

1. (a) Evaluate $\int \frac{\sqrt{\arcsin(x)}\, dx}{\sqrt{1-x^2}}$
   (b) Evaluate $\int_{\sqrt{2}/3}^{2/3} \frac{dy}{|y|\sqrt{9y^2-1}}$

6. [10 points each] Write out definite integrals that give the volumes of **both** of the following. **Do not evaluate the integrals.**

1. (a) The solid:
      i. with base the region in the xy-plane between the curve $y = 4\sin(x)$ and the interval $[0, \pi]$ on the x-axis
      ii. with cross-sections perpendicular to the x-axis that are squares with one side running from the x-axis to the curve.
   (b) The solid obtained by revolving the region in the first quadrant bounded by $y = x^2$ and $y = 2x$ about the vertical line $x = 3$.