Mathematics 181B **Spring 2012**

Exam 2

February 02, 2012

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Name

Technology used: ____ **Only write on one side of each page.**

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

"*Drawing on my fine command of the English language, I said nothing.*" — Robert Benchley

Problems

1. [20 points] Find the volume of the following solid (this is **not** a solid of revolution). The base of the solid is the region in the xy-plane bounded by the parabola $x^2 = 4y$ and the line $y = 1$.Each cross section perpendicular to the x-axis is an equilateral triangle with base in the xy-plane.

2. [20 points] Find the length of the curve given by $x = (y^3/6) + 1/(2y)$ from $y = 2$ to $y = 3$.

3. [15 points each] Evaluate **three** (3) of the following integrals:

   (a) $\int \sin(\ln(x))\ dx$

   (b) $\int \cos^5(x)\sin^4(x)\ dx$

   (c) $\int_0^{\ln(4)} \frac{e^t}{\sqrt{e^{2t}+9}}\ dt$ (use a simplifying substitution and then a trigonometric substitution)

   (d) $\int \frac{1}{y+\sqrt{y}}\ dy$ (hint: first make a substitution that removes the square root)

4. [15 points] Do **one** (1) of the following.

   (a) The region bounded by the half-circle with equation $x = \sqrt{9 - y^2}$ and the y-axis is rotated around the y-axis to form a solid ball. A hole of radius 1 (diameter 2) centered on the y-axis is then bored through the ball (see the figure on the board). Use the washer method (method of slicing) to set up, but **do not evaluate**, definite integral(s) that give the volume remaining in this "cored" solid ball.

   (b) The area of a washer formed by concentric circles centered at the origin and of radius x and $x + \Delta x$ (see figure on the board) is approximately $2\pi x \cdot \Delta x$. Use this estimate and the Riemann sum process to develop a definite integral that represents the area contained in a circle of radius R.

   Start by partitioning the interval $[0, R]$ on the x-axis into n subintervals of equal length and go through the entire Riemann sum development process.

5. **Extra Credit**. No matter how much one studies and how much one learns, there is always some question you wish you had asked. I will give 5 points of extra credit for a thoughtful question of this type.

$\frac{\sqrt{3}}{2}\int_0^2 \left(1 - \frac{1}{4}x^2\right)^2 dx = \frac{8}{15}\sqrt{3} = 0.923\,76$