Mathematics 181D **Spring 2008**

Exam 2

February 28, 2008

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Name

Technology used: ____ **Only write on one side of each page.**

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

Do this problem

1. Set up, **but do not evaluate,** definite integrals for **two** (2) of the following.

   (a) An integral representing the volume of the solid with base the region in the xy-plane bounded by the x-axis and the graph of the curve $y = \sqrt{16 - x^2}$ and with cross sections perpendicular to the x-axis that are semicircles with diameter lying in the xy-plane.

   (b) An integral representing the volume of the solid or revolution obtained by revolving the region bounded by the graphs of $y = 4 - x^2$ and $y = 2 - x$ about the line $y = -2$.

   (c) An integral representing the volume of the solid or revolution obtained by revolving the region bounded by the graphs of $y = 4 - x^2$ and $y = 2 - x$ about the line $x = 3$.

Do any four (4) of the following problems

1. Find the area of the region bounded by the graph of $y = \frac{\pi}{2}\cos(x)\left[\sin(\pi + \pi\sin(x))\right]$ over the interval -$\pi \leq x \leq -\frac{\pi}{2}$. Note that $y = 0$ at $x = -\pi$ and $x = -\pi/2$.

2. Find the length of the curve given by the parametrization $x = \frac{1}{3}(2t+3)^{3/2}$, $y = t + \frac{1}{2}t^2$, $0 \leq t \leq 3$.

3. Suppose that a cup of soup cooled from $90°C$ to $60°C$ after 10 minutes in a room whose temperature was $20°C$. Use Newton's law of cooling to determine how much longer it would take for the soup to cool to $35°C$.

4. A certain population of sheep (where y is measured in thousands of sheep) is modeled by the differential equation $\frac{dy}{dt} = 2y(1-y)$. Show that the function displayed below solves this separable differential equation. [Note that you are not being asked to solve the differential equation.]

$$y(t) = \frac{1}{1 + e^{-2t}}$$

5. For this problem pretend that you know the formula for the area inside a circle of radius R is πR^2 but that **you do not know the formula for the circumference of a circle.** Let $C(x)$ be the notation for the function that gives the circumference of a circle of radius x.

   Draw a picture in which you partition the interval $0 \leq x \leq R$ and then use your understanding of Riemann sums to build a definite integral, with integrand involving $C(x)$, that equals the area inside a circle of radius R. Briefly explain your integral formula but do not evaluate this integral.