Mathematics 181B **Spring 2012**

Exam 1

February 02, 2012

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Name

Technology used: ____ **Only write on one side of each page.**

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

1. [15 points] Without using a calculator, evaluate the following indefinite integral

$$\int \left(\frac{1}{|x|\sqrt{x^2-1}} + \frac{2}{x^2+1} - \frac{3}{\sqrt{1-x^2}} + \frac{4}{x} - e^x + \sin(x) - \sec^2(x) + \sec(x)\tan(x) - x^{-5/3} \right) dx$$

2. [15 points] If we use the partition points $x_0 < x_1 < x_2 < \cdots < x_n$ to partition the interval $[2, 5]$ into n subintervals of equal length

   (a) What is the value of Δx in terms of the letter n?

   (b) Write the values of x_0, x_1, x_2, x_k, and x_n in terms of the letter n.

   (c) Use sigma notation to write, in terms of the letter n, the Riemann sum for the function $f(x) = 2\pi\sqrt{x}$ that uses the **left endpoint** of each subinterval as the value of c_k.

3. [15 points] Do **ONE** (1) of the following.

   (a) If we partition the interval $[0, 2]$ into n subintervals of equal width and select c_k as the right endpoint of each subinterval, then the corresponding Riemann sum for the function $f(x) = 8 - x^3$ is $\sum_{k=1}^{n} \left(8 - \left(\frac{2k}{n}\right)^3\right) \frac{2}{n}$.

   Use the fact that $f(x) = 8 - x^3$ is monotone decreasing over the interval $[0, 2]$ to find an error bound for this estimate. Include any pertinent figures and write your answer as a function of n (the number of subintervals).

   (b) Express the following limit as a definite integral. Do not evaluate the limit. [Note that $\Delta x = \frac{5}{n}$.]

$$\lim_{\|P\| \to 0} \sum_{k=1}^{n} \left[9\left(2 + \frac{5k}{n}\right)^5 - \left(2 + \frac{5k}{n}\right)^2 + 15 \right] \frac{5}{n}$$

4. [10 points] Do **ONE** (1) of the following.

   (a) Find the derivative of $F(x) = \int_{x^2}^{x^3} \sqrt[3]{\cos(4t)}\, dt$.

   (b) Find a function f that satisfies the equation

$$\sec(x) = \int_2^x \sqrt{4 + f(t)}\, dt.$$

5. [15 points each] Use substitution to evaluate **TWO** (2) of the following indefinite integrals.

(a)

$$\int \frac{4x\sqrt{\arcsin\left(x^2\right)}}{\sqrt{1-\left(x^2\right)^2}}\,dx$$

(b)

$$\int_0^{\ln(9)} e^{\theta}\left(e^{\theta}-1\right)^{1/2}\,d\theta$$

(c)

$$\int \frac{3\sin(x)\cos(x)}{\sqrt{1+3\sin^2(x)}}\,dx$$

6. [15 points] Solve the initial value problem

$$\frac{d^2y}{dx^2}=\frac{1}{(x-2)^2},\quad y'(3)=0,\quad y(3)=5$$