Mathematics 122 – **B** **Fall 2004**

November 23 **Exam 4**

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Name

Technology used:______________________________ Directions:

Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

Problems

1. (10 points each) Evaluate any **two** (2) of the following integrals.

   (a) $\int \frac{x^3+4}{x(x^2+4)}\ dx$

   (b) $\int \frac{dx}{(x^2+9)^{3/2}}$

   (c) $\int \sec^5(x)\tan^3(x)\ dx$

2. (15 points) Evaluate **one** (1) of the following improper integrals.

   (a) $\int_1^e \frac{dx}{x\sqrt[4]{\ln(x)}}$

   (b) $\int_0^4 \frac{2x-1}{x-3}\ dx$

3. (10 points each) Choose any **four** (4) of the following infinite series and determine if they converge or diverge. If a series that converges can be summed by using discrete antiderivatives, use the formulas in the "Useful Information" section to do so.

   (a) $\sum_{k=1}^{\infty}\left[2(0.1)^k + (0.2)^k\right]$

   (b) $\sum_{k=17}^{\infty} k^{-4}$

   (c) $\sum_{k=17}^{\infty} k^{-\frac{4}{\,}}$

   (d) $\sum \frac{k^2}{5k^2+2}$

   (e) $\sum \frac{k}{k^2+1}$

4. (15 points) Using discrete antiderivatives we can show that $\sum_{k=4}^{\infty}\frac{1}{k(k-1)(k-2)} = \sum_{k=4}^{\infty}\frac{24}{k^3} = 2$ (**do not show this**).

   Use the fact that $0 < \frac{1}{k^3} < \frac{1}{k(k-1)(k-2)}$ for each $k = 4, 5, 6, \cdots$ and the Bounded Monotone Convergence Theorem to show that the discrete antiderivative sequence $A(n) = \sum_{k=4}^{n-1}\frac{1}{k^3}$converges to a number less than or equal to 2.

Take Home Portion:

Do not work with anyone on these problems. However, you may use the textbook and your notes.

Do any two of the following problems.

1. (10 points) The abbreviated table of integrals from our textbook does not have a formula for $\int \frac{du}{u^2(u^2+a^2)^2}$. Show that you understand the methods of integration in Chapter 7 by deriving the formula for the following special case where $a = 1$.

$$\int \frac{dx}{x^2\left(x^2+1\right)^2}.$$

2. (10 points) This problem is in two parts

   (a) Derive the product rule for sequences. That is, if $u(k)$ and $v(k)$ are sequences with domain $k = 1, 2, 3, \cdots$, then

   $$D_k\left[u(k)\,v(k)\right] = D_k\left[u(k)\right]v(k+1) + u(k)\,D_k\left[v(k)\right].$$

   (b) Mimic the method used to derive the integration by parts formula to derive the "summation by parts" formula. That is, if $u(k)$ and $v(k)$ are sequences with domain $k = 1, 2, 3, \cdots$, then

   $$\sum_{k=1}^{n-1} u(k)\,D_k\left[v(k)\right] = u(k)\,v(k)\Big|_{k=1}^{n} - \sum_{k=1}^{n-1} v(k+1)\,D_k\left[u(k)\right].$$

3. (10 points) Use the summation by parts formula to determine whether or not the following infinite series converges. If the series converges determine the value it converges to.

$$\sum_{k=1}^{\infty} \frac{2k-1}{3^k}.$$