Mathematics 122 – AB Fall 2004

November 2 **Exam 3**

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Name

Technology used: _____ Directions:

Be sure to show all steps in your solutions. Partial credit is based on your work – not on your answer. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

1. (10 points each) Set up **but do not evaluate** three (3) of the following four problems. Use the methods of Cavalieri and Cylindrical Shells at least once each. Clearly indicate which three problems are to be graded.

   (a) The base of a solid is bounded by the x-axis and the graph of the curve $y = 9 - x^2$. Cross sections of the solid perpendicular to the x-axis are semicircles. Find the volume of the solid.

   (b) Find the volume of the solid obtained when the region bounded by the graphs of the curves $y = \sqrt{x}$ and $y = x$ is rotated about the line $x = -1$.

   (c) The disk enclosed by the circle $x^2+y^2 = 4$ is revolved about the y-axis to generate a solid sphere. A hole of diameter 2 is then bored through the sphere along the y-axis. Find the volume of the "cored" sphere.

   (d) Find the volume of the solid obtained by rotating the region bounded by the graph of $y = e^{-x^2}, \quad x \geq 0$ about the y-axis.

2. (10 points each) Set up **but do not evaluate** three (3) of the following five problems.

   (a) Find the surface area of the figure generated by rotating the graph of $y = \sqrt{x}$, $0 \leq x \leq 4$ around the x-axis.

   (b) Find the arclength of the graph of $f(x) = 1/3x^{3/2} - x^{1/2}, 1 \leq x \leq 4$.

   (c) Set up an integral that represents the length of the curve $y = \ln\left(1 - x^2\right)$ from $x = 0$ to $x = \frac{1}{2}$.

   (d) The work done in compressing a spring 2 inches from its natural length is 10 foot-pounds. What is the work done in compressing the spring an additional two inches? [Hooke's Law for the force function associated with a spring is $F(x) = kx$ where k is a constant and x is the length the spring is stretched (compressed) from it's natural length.]

   (e) Show that the surface area of a sphere of radius R is $SA = 4\pi R^2$.

3. (8 points each) **Evaluate** five (5) of the following integrals. Clearly cite any formula you use from the table of integrals.

   (a)
   $$\int \sin^4(x)\cos^5(x)\,dx$$

   (b)
   $$\int \sec^3(4x)\,\tan^3(4x)\,dx$$

(c)

$$\int x^3 \sec^2(x^2) \ dx$$

(d)

$$\int x \ln(x+2) \ dx$$

(e)

$$\int \frac{e^{3x}dx}{\sqrt{1+e^{3x}}}\, dx$$

(f)

$$\int \frac{e^{4x}dx}{1+e^{8x}}$$

(g)

$$\int \frac{7}{\sqrt{x^2+4x-5}}\, dx$$

(h)

$$\int \frac{dx}{1+\cos(x)}$$