# Mathematics 122 – A

**Fall 2004**

**Exam 2**

**October 5, 2004**

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*Name*

**Technology used:**____________________________________________ Directions:

Be sure to show all steps in your solutions. Partial credit is based on your work – not on your answer. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

## The Problems

1. ( 15 points ) Do **one** (1) of the following

   (a) Match the slope fields on the accompanying sheet with the differential equations

      i. $y' = y\cos(y)$ : ________________

      ii. $y' = x\cos(y)$ : ________________

      iii. $y' = y\cos(x)$ : ________________

   (b) On May 7, 1992, the space shuttle *Endeavor* was launched on mission STS-49. The table below, provided by NASA, gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters. Use the data in the table and Simpson's Rule to estimate the height above the earth's surface of the space shuttle *Endeavor*, 62 seconds after liftoff. $\left[S_n = \frac{1}{3}(M_n + 2T_n) = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)\right)\right]$

| Event | Time (s) | Velocity (ft/s) |
|---|---|---|
| Launch | 0 | 0 |
| Begin roll maneuver | 10 | 185 |
| End roll maneuver | 15 | 319 |
| Throttle to 89% | 20 | 447 |
| Throttle to 67% | 32 | 742 |
| Throttle to 104% | 59 | 1325 |
| Maximum dynamic pressure | 62 | 1445 |
| Solid rocket booster separation | 125 | 4151 |

   (c) Given the sequence $a(k) = 3k+1,\ k = 1, 2, \cdots$, Use our techniques for 'discrete derivatives' and discrete antiderivatives' to find a formula for the general term of the discrete antiderivative $A(k)$ of $a(k)$ that satisfies $A(1) = 5$.

2. ( 15 points each) Do any **two** (2) of the following

   (a) Find the orthogonal trajectories of the family of functions given by $y = x^4 + C$.

   (b) Use separation of variables to find the general solution of

$$y'(x) = \frac{x^2}{\cos(y(x))}.$$

   Then find the specific solution that satisfies the initial condition $y(2) = \frac{\pi}{2}$.

(c) Show that the function $y = \frac{1}{x}\int_1^x \frac{e^t}{t}\,dt$ is a solution of the differential equation $x^2y' + xy = e^x$.

3. ( 15 points ) Do **one** (1) of the following.

(a) Find a function $f$ that satisfies the equation

$$\cos(x) + x^2 = \int_2^x \sqrt{3t^2 + tf(t)}\,dt.$$

[Hint: take the derivative of both sides of the equation.]

(b) If $F(x) = \int_1^x f(t)\,dt$ where $f(x) = \int_1^{x^2} \frac{\sqrt{1+u^2}}{u}\,du$, find $F''(2)$.

4. ( 5 points each) Evaluate **five** (5) of the following indefinite and definite integrals. Use the First Fundamental Theorem of Calculus and substitution.

(a)

$$\int \frac{1}{x-5}\,dx$$

(b)

$$\int \frac{23}{\sqrt{1-x^2}} - \sec(x)\tan(x) \quad dx$$

(c)

$$\int \frac{dx}{x\ln(x)}$$

(d)

$$\int \frac{\arctan(2x)}{1+4x^2}\,dx$$

(e)

$$\int \frac{(3x+6)\,dx}{(x^2+2x-3)^{\frac{2}{3}}}$$

(f)

$$\int_0^{0.5} \frac{dx}{\sqrt{1-x^2}}$$

(g)

$$\int \frac{5-2x}{\sqrt{1-x^2}}\,dx$$

5. ( 15 points ) Do any **one** (1) of the following.

(a) Given the function $f(x) = e^{x^2}$.

i. Show the work justifying $f''(x) = (2+4x^2)\,e^{x^2}$.

ii. What is the smallest value of the integer $n$ necessary to guarantee that the Trapezoid rule will approximate the integral

$$\int_0^1 e^{x^2}\,dx$$

to within $\epsilon = 10^{-16}$? $[E_n \leq \frac{1}{12}\frac{(b-a)^3}{n^2}M]$

(b) Most of us know the number $e$ is approximately 2.72. Use the fact that $e - 1 = \int_0^1 e^x\,dx$ and the error bound for Simpson's Rule, $E_n$, to determine the number of subintervals necessary to determine the value of $e$ accurate to within $10^{-9}$. $[E_n \leq \frac{1}{180}\frac{(b-a)^5}{n^4}M]$

(c) Explain why Simpson's Rule with $n = 2$ would give an exact answer (no error) for the integral $\int_0^{100} (2x^3 - 5x^2 + 23)\,dx$. [Look carefully at the error bound formula.]

**Slope Fields for Problem 1.**

A.

B.

C.

D.