Mathematics 121-A   Spring 2000   Exam 6

April 14, 2000

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Name

Technology used: _______________________

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

1. (20 points) On the attached sheet of graph paper, sketch the graph of the function f that satisfies the following conditions.

Points on the graph of f	$(-4,-1), (-3,1), (-1,-2), (0,0), (3,2), (4,4)$
Inputs where $f'(x)$ Does Not Exist	$x=-4,\ x=-1$
Inputs where $f'(x)=0$	$x=-3,\ x=3$
Intervals where $f'(x)>0$	$(-\infty,-3), (-1,3)$
Intervals where $f'(x)<0$	$(-3,-1)$
Intervals where $f''(x)>0$	$(-\infty,-4), (3,4)$
Intervals where $f''(x)<0$	$(-4,-1), (-1,3), (4,\infty)$
Limit information	$\lim_{x\to-4} f'(x)=\infty$
	$\lim_{x\to-1^-} f'(x)=-\infty$
	$\lim_{x\to-1^+} f'(x)=2$

2. (15 points) Find the absolute maximizers, minimizers, maximum and minimum of $f(x)=x^{2/3}$ on $[-1,8]$ or show they do not exist.

3. Given the function $f(x)=-x^3+3x^2-1$.

   (a) (10 points) Find the absolute maximum of f on $[0,\infty)$.

   (b) (10 points) Use calculus to carefully explain why this is an **absolute** maximum.

4. (20 points) Given the function $f(x)=\left(x^2-1\right)^2$.

   (a) Find all critical points of f.

   (b) Find all second order critical points of f.

   (c) Determine all intervals where f is strictly increasing and all intervals where f is strictly decreasing.

   (d) Determine all intervals where f is concave up and all intervals where f is concave down.

   (e) Classify the critical points as local maximizers, local minimizers or neither.

5. (15 points) Use Rolle's Theorem or the Mean Value Theorem to show that if f is a polynomial with at least three zeros, say $f(x_1)=f(x_2)=f(x_3)=0$, then there must be a point c at which $f''(c)=0$.