

February 11, 2000

NameTechnology used:

Directions: Be sure to show all of your work. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

1. (15 points) Do **one** of the following.
 - (a) A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 10 cm by 22 cm by cutting out equal squares of side x at each corner and then folding up the sides. Express the volume V of the box as a function of x . Include the domain of your function.
 - (b) A closed box with a square base is to have a volume of 300 cubic feet. The material for the top and bottom of the box costs \$2.00 per square foot and the material for the sides costs \$1.00 per square foot. Express the cost of building the box as a function of the length of its base. Include the domain of your function.
2. (15 points each) Do **two** of the following.
 - (a) Suppose the graph of a function f is given. Write equations for the graphs that are obtained from the graph of f as follows.
 - i. Shift 3 units upward and 4 units to the left.
 - ii. Reflect about the y -axis then stretch vertically by a factor of 2.
 - (b) Graph $f(x) = \arcsin(2x)$, not by plotting points or using your calculator, but by starting with the graph of $g(x) = \sin(x)$ and then applying the appropriate transformations.
 - (c) Write the equation for the graph that is obtained by reflecting the graph of $y = x^3$ about the line $x = 2$.
3. (15 points) Find the exact value of
 - (a) $\sin\left(\arccos\left(\frac{\sqrt{5}}{4}\right)\right)$
 - (b) $\sec(\arctan(x))$
4. (15 points) Given $f(x) = \frac{x+7}{x-5}$, $x \neq 5$. Find f^{-1} , if it exists.
5. (15 points) Evaluate the following limits or show they do not exist.
 - (a)

$$\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{3x^2 + 5x - 7}$$

(b)

$$\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$$

6. (10 points) Provide an $\varepsilon - \delta$ proof that $\lim_{x \rightarrow 3} \frac{2x^2 - 7x + 3}{x - 3} = 5$.