# Mathematics 121                        Spring 2006

**Algebra Review Quiz**

**January 18, 2006**

<p style="text-align:right">_____<br><i>Name</i></p>

**Directions:**     This quiz covers a number of algebraic concepts and techniques that we will use frequently this semester. It should give you a reasonable idea of what algebra skills you should review.
Without using a calculator, do as many of these as you can in the time provided.

## The Problems

1. Do you know the names and/or greek symbols for the following:

   (a) $\beta$ is Beta

   (b) $\Sigma$ is a capitalized Sigma

   (c) alpha $= \alpha$

   (d) theta $= \theta$

2. Give the exact values of the first three and complete the identities of the last two of the following:

   (a) $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

   (b) $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

   (c) $\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$

   (d) $\arctan\left(1\right) = \frac{\pi}{4}$

   (e) $\sin\left(A + B\right) = \sin\left(A\right)\cos\left(B\right) + \cos\left(A\right)\sin\left(B\right)$

   (f) $\frac{\sec(x)}{\tan(x)} = \frac{1}{\sin(x)} = \csc\left(x\right)$

3. Simplify the following:

   (a)
   $$6 + x - 2\left(3 - 4\left(x - y\right)\right) = 9x - 8y$$

   (b)
   $$(x + 2)\left(x^2 - 2x + 4\right) = x^3 + 8$$

   (c)
   $$\frac{x^2 - 9}{x^2 - 4x - 21} = \frac{x - 3}{x - 7}$$

   (d)
   $$\frac{x^3 - 4x}{x^{1/2}} = x^{1/2}\left(x^2 - 4\right)$$

4. Factor the following:

(a)

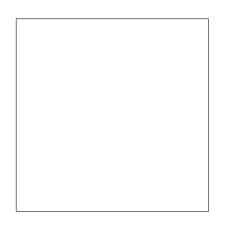$$x^2 - (x-3)^2 = 3(2x-3) = 6x - 9$$

(b)

$$x(x-1)^{-1/2} + 2(x-1)^{1/2} = (x-1)^{-1/2}(3x-2)$$

5. Which of the following are equal to $\sqrt{x^2 + 7^2}$ **The answer is e. — None of the above.**

   (a) $x + 7$
   (b) $|x| + 7$
   (c) $|x + 7|$
   (d) all of the above
   (e) none of the above

6. If the rule for a function f is given by $f(x) = 2x^2 + 3x$, evaluate and simplify $f(x+h) - f(x)$.

$$h(2h + 4x + 3)$$

7. Find the set of all real numbers $x$ that satisfy the following inequalities. Write your solutions in both set notation and interval notation.

   (a) $\{x : -3 \le 3 - 2x \le 5\} = \{x : -1 \le x \le 3\} = [-1, 3]$
   (b) $\{x : x^2 + 5x - 14 < 0\} = \{x : -7 < x < 2\} = (-7, 2)$
   (c) $\{x : |x - 5| < 2\} = \{x : 3 < x < 7\} = (3, 7)$
   (d) $\{x : |7 - 2x| < 5\} = \{-1 < x < 6\} = (-1, 6)$

8. Sketch the graph of $(x-1)^2 + (y+2)^2 = 4$.

   This is a circle of radius 2 centered at the point $(1, -2)$.

9. Where do the graphs of $y = x^2$ and $y = x + 12$ intersect? — $\{x : x = -3 \text{ or } x = 4\}$.

10. If an isosceles triangle with two sides equal to $x$ has a perimeter of 100, write the area of the triangle as a function of $x$. — $A = 25\sqrt{x^2 - 2500}$.

11. Find the distance between the points $(2, -1)$ and $(3, 5)$. — $\sqrt{37}$.

12. What is the relationship between the graphs of the following functions?

    (a) $y = f(x)$ and $y - k = f(x)$ — the second graph is the first graph shifted $k$ units vertically.

    (b) $y = f(x)$ and $y = f(x - h)$ — the second graph is the first graph shifted $h$ units horizontally.

    (c) $y = f(x)$ and $y = -4f(x)$ — the second graph is the result of taking each point of the first graph and plotting it on the other side of and four times as far from the $x$-axis. In other words, flip the graph around the $x$-axis and stretch it vertically by a factor of 4.

    (d) $y = f(x)$ and $y = f(3x)$ — the second graph is the result of taking each point of the first graph and plotting it $1/3$ as far from the $y$-axis. In other words, compress the graph horizontally by a factor of 3.

13. If $f(x) = \sin(x)$ and $g(x) = 3x - 7$, what are the following?

    (a) $(f + g)(x) = \sin(x) + 3x - 7$

    (b) $(3f - 4g)(x) = 3\sin(x) - 12x + 28$

    (c) $\left(\frac{2f}{g}\right)(x) = \frac{2\sin(x)}{3x-7}$

    (d) $(f \circ g)(x) = f(g(x)) = \sin(3x - 7)$

    (e) $(g \circ f)(x) = 3\sin(x) - 7$

14. Explain why the answer to the following question is **not yes**. "Is $-x$ a negative number?" — It depends on the value of $x$. If $x$ is already negative then $-x$ is positive.

15. Explain why the following is **not correct.**

$$\frac{(x-7)(x^2+3) - (x^3+x)}{(x-7)(x+1)} = \frac{(x^2+3) - (x^3-x)}{(x+1)}.$$

    — The problem is that there is no $(x-7)$ in the second term of the numerator on the left hand side. The result would be correct if the numerator of the left hand side had been $(x-7)(x^2+3) - (x-7)(x^3+x)$. Then, since both terms would have had a factor of $(x-7)$, it could have been factored out and cancelled with the one in the denominator.

16. Explain why $\sqrt{49} \neq -7$. — $\sqrt{A}$ means the **positive** square root of $A$. So $\sqrt{A}$ can **never** be negative. In symbols, the definition of $\sqrt{x^2}$ is $\sqrt{x^2} = |x|$.

17. Does $-9^2$ equal $+81$ or $-81$? — The answer is $-81$. In order to get $+81$ we would need parentheses: $(-9)^2 = 81$.

18. Take a look at the website http://www.math.vanderbilt.edu/~schectex/commerrs/ for a discussion of many other common mathematical errors.