Mathematics 121-A **Spring 2006**

Exam 5 and Cumulative Final Exam

May 8, 2006

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Name

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Show your work: answers that can be obtained from a calculator will not receive credit.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Examination 5

Do any five (5) of the following.

1. (20 points) Evaluate any four (4) of the the following indefinite integrals.

   (a)
   $$\int \frac{1}{7x}\, dx$$

   (b)
   $$\int \left(2e^x + 3\sec(x)\tan(x)\right)\, dx$$

   (c)
   $$\int \left(\sec^2(x) + \frac{1}{x}\right)\, dx$$

   (d)
   $$\int \frac{1}{t^2}\left(\frac{5}{t^3} + \frac{2}{t}\right)\, dt$$

   (e)
   $$\int \frac{1}{\sqrt{9 - 9x^2}}\, dx$$

   (f)
   $$\int 4\left(\cos(x)\right)^3\left(-\sin(x)\right)\ \ dx$$

2. (20 points) Do **both** of the following.

   (a) Use the First Fundamental Theorem of Calculus to find the exact area of the region bounded by the graph of $y = x^3 + 3x^2$,the x- axis, and the vertical lines $x = 1$ and $x = 4$.

(b) Find the exact area of the region below the graph of $y = x^3 + 3x^2$,above the graph of $y = 2x^2 - x$ and between the vertical lines $x = 1$ and $x = 4$.

3. (20 points) On May 7, 1992, the space shuttle *Endeavor* was launched on mission STS-49. The table below, provided by NASA, gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters. Use the data in the table to form a Riemann sum that estimates the height above the earth's surface of the space shuttle *Endeavor*, 62 seconds after liftoff. **Simplify your sum.**

Event	Time (s)	Velocity (ft/s)
Launch	0	0
Begin roll maneuver	10	185
End roll maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
Solid rocket booster separation	125	4151

4. (20 = 6 + 7 + 7 points) The following limit gives the exact area of a region in the plane as the limit of Riemann sums of a function f. The Riemann sums were obtained by partitioning an interval $[a, b]$ into n subintervals of equal size Δx and using the right endpoints of each subinterval. **DO NOT EVALUATE** the limit.

$$\lim_{n\to+\infty} \sum_{k=1}^{n} \left[\left(3 + \frac{4}{n}k\right)^5 - 2\left(3 + \frac{4}{n}k\right)^2 + 5 \right] \frac{4}{n}.$$

(a) What is Δx ?

(b) What is the function f ?

(c) What is the interval $[a, b]$?

5. (20 points) Use the Second Fundamental Theorem of Calculus to answer both of the following.

(a) Find the derivative $G'(x)$, of

$$G(x) = e^x \int_3^x \frac{\tan(t)}{t^2 + 1}\, dt.$$

(b) Find the derivative $H'(x)$ of

$$H(x) = \int_1^{e^{3x}} \ln\left(t^2 + 1\right) dt.$$

6. (20 = 8 + 6 + 6 points) The slope $F'(x)$ at various points on a graph is given in the figure below. Each block of the grid is 2 units wide and 2 units high. Use this information to sketch the graph of the antiderivative F of F' that passes through the point $(0, -1)$. **Draw your sketch on this graph and record your answers here.**

(b) What is the value of $F'(6)$?

(c) Use your graph to estimate the value of $F(6)$?

(a)

Figure 1:

Cumulative Portion of Exam

Do any five (5) of the following.

1. (20 points) Find the derivatives of the following functions. You may use any rules or formulas. **Do Not simplify your answers**.

   (a)
   $$y = \left(x^{1/2} + x\right)\cos(x)$$

   (b)
   $$f(x) = e^{\tan\left(x^2+7\right)}$$

   (c)
   $$f(x) = \frac{\ln(\sec(x))}{\sin(x)}$$

   (d)
   $$y = x^2 \arctan(x)$$

   (e) Find $F'(1)$ where
   $$F(x) = e^{f\left(x^3-x\right)}$$
   and the following table gives the only known outputs for the functions f and f'.

x	f	f'
0	-3	2
1	5	3
2	1	4

2. (20 points) Use an ε - δ argument to **prove** that
   $$\lim_{x\to 3} \frac{x^2-9}{x-3} = 6.$$

3. (20 points) **Use the definition** of derivative (that is, $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$) to compute $f'(x)$ if $f(x) = 2x^2 - 3x$.

4. (20 points) An 1125 cubic foot open-top rectangular tank with a square base x feet on a side and y feet deep is to be built with its top flush with the ground to catch runoff water. The steel for the tank costs \$ 5.00 per square foot and the cost of excavating the hole is $10xy$ dollars. Find the minimum possible cost of the tank.

5. (20 points) Do **one** (1) of the following related rates problems.

   (a) Oil is leaking from an ocean tanker at the rate of 5000 cubic meters per minute resulting in a circular oil slick of (average) depth 4 cm. How fast is the radius increasing 4 hours (= 14400 seconds) after the leaking began? Give your answer to the nearest hundredth of a meter per second. **[Make sure your units are consistent].**

   (b) (A Calculus classic) The bottom of a 10-foot ladder is being pulled away from a wall at $dx/dt = 2$ feet per second. How fast is the top going down the wall when the top is 6 feet above the ground?

6. (20 points) Draw a **careful** sketch over the domain $-5 \leq x \leq 10$ of a function f that satisfies all of the following:

   (a) **Facts about** f :

      i. The points $(-3,-2)$, $(-1,0)$, $(0,2)$, $(2,0)$, $(5,-2)$, and $(7,-6/5)$ are on the graph.
      ii. The line $x = 1$ is a vertical asymptote.
      iii. $\lim_{x\to-\infty} f(x) = \infty$
      iv. $\lim_{x\to\infty} f(x) = -1$
      v. $\lim_{x\to 1} f(x) = \infty$

   (b) **Facts about** f' :

      i. $f'(x) = 0$ only when $x = 5$.
      ii. $f'(x)$ Does Not Exist when $x = -3$ and $x = 2$.
      iii. $f'(x) > 0$ on the intervals: $-3 < x < 1$ and $x > 5$
      iv. $f'(x) < 0$ on the intervals: $x < -3$ and $1 < x < 5$
      v. $\lim_{x\to 1^-} f'(x) = \infty$
      vi. $\lim_{x\to 1^+} f'(x) = -\infty$
      vii. $\lim_{x\to -3^-} f'(x) = -1/2$
      viii. $\lim_{x\to -3^+} f'(x) = -\infty$

   (c) **Facts about** f''

      i. There is a point of inflection at $(7,-6/5)$
      ii. $f''(x) > 0$ on the intervals: $-3 < x < 1$ and $1 < x < 7$
      iii. $f''(x) < 0$ on the intervals: $x < -3$ and $x > 7$