

February 11, 2000

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NameTechnology used: 

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**Directions:** Be sure to show all of your work. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

**The Problems**

1. ( 15 points) Do **one** of the following.
  - (a) A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 10 cm by 22 cm by cutting out equal squares of side  $x$  at each corner and then folding up the sides. Express the volume  $V$  of the box as a function of  $x$ . Include the domain of your function.
  - (b) A closed box with a square base is to have a volume of 300 cubic feet. The material for the top and bottom of the box costs \$2.00 per square foot and the material for the sides costs \$1.00 per square foot. Express the cost of building the box as a function of the length of its base. Include the domain of your function.
2. ( 15 points each) Do **two** of the following.
  - (a) Suppose the graph of a function  $f$  is given. Write equations for the graphs that are obtained from the graph of  $f$  as follows.
    - i. Shift 3 units upward and 4 units to the left.
    - ii. Reflect about the  $y$ -axis then stretch vertically by a factor of 2.
  - (b) Graph  $f(x) = \arcsin(2x)$ , not by plotting points or using your calculator, but by starting with the graph of  $g(x) = \sin(x)$  and then applying the appropriate transformations.
  - (c) Write the equation for the graph that is obtained by reflecting the graph of  $y = x^3$  about the line  $x = 2$ .
3. ( 15 points) Find the exact value of
  - (a)  $\sin\left(\arccos\left(\frac{\sqrt{5}}{4}\right)\right)$
  - (b)  $\sec(\arctan(x))$
4. ( 15 points) Given  $f(x) = \frac{x+7}{x-5}$ ,  $x \neq 5$ . Find  $f^{-1}$ , if it exists.
5. ( 15 points) Evaluate the following limits or show they do not exist.
  - (a)

$$\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{3x^2 + 5x - 7}$$

(b)

$$\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$$

6. ( 10 points) Provide an  $\varepsilon - \delta$  proof that  $\lim_{x \rightarrow 3} \frac{2x^2 - 7x + 3}{x - 3} = 5$ .