10.3 Homework

Four or five of you solved this problem by using geometry. Another ten or so used components. Both of these methods are fine for this homework assignment but you should make sure you understand how to use vector algebra without components to solve such problem.

Here is a quick outline that only uses the algebraic properties of dot products and the fact that $|\vec{u}|$ and $|\vec{v}|$ are both equal to the circle radius and hence they equal each other.

$$\begin{aligned} (\vec{u} - \vec{v}) \cdot (-\vec{u} - \vec{v}) &= (\vec{u} - \vec{v}) \cdot (-\vec{u}) + (\vec{u} - \vec{v}) \cdot (-\vec{v}) \\ &= -(\vec{u} - \vec{v}) \cdot \vec{u} - (\vec{u} - \vec{v}) \cdot \vec{v} \\ &= -\vec{u} \cdot \vec{u} + \vec{v} \cdot \vec{u} - \vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{v} \\ &= -|\vec{u}|^2 + \vec{u} \cdot \vec{v} - \vec{u} \cdot \vec{v} + |\vec{v}|^2 \\ &= -|\vec{u}|^2 + |\vec{v}|^2 \\ &= 0. \end{aligned}$$