Problems: Total from volume density

1. Relative to a chosen cartesian coordinate system, a solid object sits in the first octant bounded by $z = 4 - x^2 - y$ and the coordinate planes. The object has a non-uniform composition so that the volume mass density is given by $\delta(x, y, z) = 3z$. Compute the total mass of the solid.

   Answer: $M = \dfrac{1024}{35}$

2. A solid (right circular) cylinder of radius R and height H has a non-uniform composition so that the volume mass density is proportional to the distance from the lateral surface reaching a maximum δ_0 along the central axis. Compute the total mass M.

   Answer: $M = \dfrac{1}{3}\pi R^2 H \delta_0$

3. Charge is distributed throughout a solid (right circular) cone of radius R and height H so that the volume charge density is proportional to the square of the distance from the vertex of the cone reaching a maximum of δ_0 along the edge of the base of the cone. Compute the total charge Q.

   Answer: $Q = \dfrac{1}{10}\pi R^2 H \dfrac{R^2 + 2H^2}{R^2 + H^2}\delta_0$

4. A solid sphere of radius R has a non-uniform composition so that the volume mass density is proportional to the distance from the center of the sphere reaching a maximum of δ_0 along the surface. Compute the total mass M. Compare this mass to the total mass for a solid sphere of the same radius having uniform composition with mass density δ_0.

   Answer: $M = \pi R^3 \delta_0$

5. A solid sphere of radius R has a non-uniform composition so that the volume mass density is proportional to the distance from the surface of the sphere reaching a maximum of ρ_0 at the center. Compute the total mass M. Compare this total mass to the total mass for a solid sphere of the same radius having uniform composition with mass density δ_0. Also, compare this total mass to the total mass for the sphere in Problem 5.