Multivariable Calculus The Basics of Writing Mathematics

- 1. Use complete sentences in paragraph form. All mathematical expressions are parts of sentences no matter if they are "in line" or displayed on a separate line.
- 2. Include simple expressions within your line of text but display complex or important expressions on a separate line.
 - The fact that 2 + 2 = 4 is simple enough to display in-line.
 - The conclusion of the Fundamental Theorem of Calculus is important enough to be highlighted in the display

$$\int_{a}^{b} f(x) dx = F(b) - F(a).$$

- Note that both of the above example sentences end with a period.
- 3. Mathematicians prefer to use the pronoun "we", active voice, and present tense in written communication. The "we" refers to the author and reader as they work together through the logic of the argument.
 - Since we are given that $g(x) = 3x^2$ we know that g'(x) = 6x.
- 4. Striking the right level of detail in your presentation can be difficult. Be aware of who your audience is and try not to overburden them with arithmetic and algebraic steps while providing enough detail to help them follow your argument. Once you decide on the mathematical level of your audience, omit algebra and arithmetic steps they should find to be routine.
- 5. For this course, you should address an audience who has the same mathematical background as you but who have not been thinking about your specific problem. For example, students in a multivariable calculus class at PLU.
- 6. Choose notation carefully. **Always** define the meaning of a new symbol before you use it in your argument. Also remember that lower and upper case letters are considered different symbols in mathematics.
 - In the following sentence we use w(t) to denote the velocity of an object whose position function is given by $s(t) = 3t^2$ where t is measured in seconds and s is measured in meters. The velocity function w has units of meters per second.
- 7. It is bad form to start a sentence with a symbol or a number. Rather than say "w'(t) = 6t.", use something like "Taking the derivative we have "w'(t) = 6t." Note that both of the above are correctly formed sentences. In particular, the symbol "=" is a verb.
- 8. Not all mathematical expressions are equations so use the words "equal" and "solve" only when working with equations. For example, we simplify the expression $\frac{x^2-5x+6}{x-2}$ in the following display but do not "solve" anything. One example is

$$\frac{x^2 - 5x + 6}{x - 2} = \frac{(x - 2)(x - 3)}{x - 2}$$

= x - 3 provided $x \neq$

2.

- 9. Be careful when you use the words "variable", "parameter", and "constant". They refer to different types of mathematical objects. Also, remember item 6 .above and be sure to define the meaning of any symbols you choose for variables, parameters and constants before you actually use them.
- 10. Avoid using a colon to lead into a displayed line of mathematics when the displayed information is needed to complete the sentence.
 - The conclusion of the Fundamental Theorem of Calculus is important enough to be highlighted in the display

$$\int_{a}^{b} f(x) dx = F(b) - F(a).$$

- 11. Label figures and tables and refer to the label in your text. Do not include the tables or figures themselves in your sentences.
- 12. Use the word "substitute" rather than "plug" in formal writing.
 - Substituting $u = x^2$ into $\int_0^2 2xe^{x^2} dx$ gives us $\int_0^4 e^u du$.
- 13. If you have a display with a string of equalities or inequalities, do not repeat the left-side of the expression.
 - Using $u = x^2$ to simplify, we obtain

$$\int_{0}^{2} 2xe^{x^{2}} dx = \int_{0}^{4} e^{u} du$$
$$= e^{u}|_{0}^{4}$$
$$= e^{4} - 1.$$