**Problems on Differentials – Solution to #1**

1. The volume $V$ of a right circular cylinder is related to the radius $r$ and height $h$ of the cylinder by $V = \pi r^2 h$.

   (a) Find the linear relation among the differentials $dV, dr, \text{and} dh$.
   **Solution**:

$$\begin{aligned} dV &= \frac{\partial f}{\partial r}\, dr + \frac{\partial f}{\partial h}\, dh \\ &= 2\pi r h\ dr + \pi r^2\ dh \end{aligned}$$

   (b) Use your result from (a) to deduce a relation among percent changes in $dV, dr, \text{and} dh$.
   **Solution**: The percent change in the volume $V$ is given by $\frac{dV}{V}$ and similarly for the other variables. Thus we have

$$\begin{aligned} \frac{dV}{V} &= \frac{2\pi r h\ dr + \pi r^2\ dh}{V} \\ &= \frac{2\pi r h\ dr + \pi r^2\ dh}{\pi r^2 h} \\ &= 2\frac{dr}{r} + \frac{dh}{h} \end{aligned}$$

   (c) If the height and the radius of a cylinder are each increased by 1%, by what percent does the volume increase?
   **Solution**: Using the result from part (b) we have

$$\begin{aligned} \frac{dV}{V} &= 2\frac{dr}{r} + \frac{dh}{h} \\ &= 2(1) + 1 \\ &= 3(\text{that is, } 3\ \% \end{aligned}$$

   (d) If the height of a cylinder is increased by 1%, how much must the radius be changed to keep volume constant?
   **Solution**: Keeping the volume constant means staying on the same level set which means $dV = 0$ giving $\frac{dV}{V} = 0$. Thus we seek $dr$ for which

$$0 = \frac{dV}{V} = 2\frac{dr}{r} + 1$$

   This gives $\frac{dr}{r} = \frac{1}{2}\%$