Problems: Total from area density

1. Charge is distributed on a flat rectangular region of dimensions L by W so that the area charge density is proportional to the distance from one corner, reaching a maximum of σ_0 at the far corner. Set up an iterated integral to compute the total charge Q. You do not need to evaluate this integral. As an optional challenge, you can try to evaluate the iterated integral.

   Answer: $Q = \dfrac{\sigma_0}{\sqrt{L^2+W^2}} \displaystyle\int_0^W \int_0^L \sqrt{x^2+y^2}\,dx\,dy$

2. Consider a thin rectangular plate of dimensions L by W with uniform thickness. The materials composing the plate vary from point to point so that the area mass density is porportional to the square of the distance from the center of the plate, reaching a maximum of σ_0 at each of the four corners. Compute the total mass M of the plate.

   Answer: $M = \dfrac{1}{3} WL\sigma_0$

3. Charge is distributed on an isosceles triangle of height H and base length B so that the area charge density is proportional to the distance from the base, reaching a maximum of σ_0 at the vertex opposite the base. Compute the total charge Q.

   Answer: $Q = \dfrac{1}{6} BH\sigma_0$

4. Charge is distributed on an equilateral triangle of side length L so that the area charge density is proportional to the square of the distance from the center of the triangle, reaching a maximum of σ_0 at each of the three vertices. Compute the total charge Q.

5. A thin disk of radius R is composed of material that varies from point to point so that the area mass density is proportional to the distance from the edge reaching a maximum of σ_0 at the center. Compute the total mass M of the disk.

   Answer: $M = \dfrac{\pi}{3} R^2\sigma_0$