Mathematics $280A, 280B$ **Fall 2012**

Hints for the exam **Exam 2**

October 11, 2012

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Name

Technology used: ______________________

Only write on one side of each page.

Show all of your work.

Calculators may be used for numerical calculations and answer checking only.

1. [10 points] Simplify the following vector $\mathbf{v}$ and express it using the unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$.

$$\mathbf{v} = (\langle 1,3,-1\rangle \cdot \langle -1,0,-2\rangle)\langle 2,1,4\rangle + (5\langle -3,-2,-1\rangle - 3\langle 1,2,3\rangle)$$

   (a) Hint: Do the dot product first: $\langle 1,3,-1\rangle \cdot \langle -1,0,-2\rangle = -1+0+2 = 1$

2. [15 points] Draw a tree diagram and write a Chain Rule formula for the derivative $\frac{\partial w}{\partial t}$ where

$$w = g(x,y,t), \quad x = h(u,v,t), \quad y = f(v,t)$$

   (a) Hint: it is easy to miss the $\frac{\partial g}{\partial t}$.

3. [8, 7 points] Do both of the following

   (a) Write an equation for the plane that is tangent to the surface $z = e^{-\left(x^2+y^2\right)}$ at the point $(0,0,1)$.

      i. Hint: Don't forget to use the chain rule. The gradient at the point is $\vec{0}$.

   (b) A certain plane passes through the point $P_0(2,3,-1)$ and the projection of the vector $\overrightarrow{OP}$ onto the normal vector of the plane is $\mathbf{i}+5\mathbf{j}-2\mathbf{k}$.Write an equation in standard form of the plane.

      i. Any non-zero vector in the direction of a normal vector is still a normal vector.

4. [8, 7 points] Do both of the following.

   (a) Articulate how gradient vectors are related to level curves/surfaces and greatest rate of change.

      i. Hint: perpendiculat to level curves at the point, in the direction of greatest rate of change of the function, magnitude **is** the greatest rate of change of the function.

   (b) Compute the directional derivative of the function $z = f(x,y) = \cos\left(xy^2\right)$ at the point $(0,0,1)$ and in the direction of the vector $\mathbf{v}$ =4$\mathbf{i}$−3$\mathbf{j}$.

      i. Hint: $D_{\vec{u}}f(P)$ requires a **unit** vector $\vec{u}$ in the direction of change.

5. [9, 6 points] Do both of the following

   (a) Around the point $(1,0)$, is $f(x,y) = x^2(y+1)$ more sensitive to changes in x or to changes in y? Why?

      i. Hint: At the point (x,y), $df = 2x(y+1)\ dx + x^2\ dy$ so at the point $(1,0)$, $df = 2\ dx + 1\ dy$

   (b) What ratio of dx to dy will make df equal 0 at $(1,0)$?

      i. Hint: Set $df = 0$ in the last equation above and solve for $\frac{dx}{dy}$

6. [15 points] Do **one** (1) of the following

   (a) Suppose $\vec{r}(t)$ is a vector-valued function with the property that for every t in the domain of $\vec{r}$, $\|\vec{r}(t)\| = 4$.
   Express $\|\vec{r}(t)\|^2$ as a dot product and take the derivative of the result to show that for every t in the domain, $\vec{r}(t)$ is orthogonal to it's derivative $\vec{r}'(t)$.

      i. Hint: Use the product rule for dot products to take the derivative of both sides of $\vec{r}(t) \cdot \vec{r}(t) = 16$.

   (b) If $\mathbf{u}_1$ and $\mathbf{u}_2$ are orthogonal unit vectors and $\mathbf{v} = a\mathbf{u}_1 + b\mathbf{u}_2$, show that $\mathbf{v} = (\mathbf{v} \cdot \mathbf{u_1})\,\mathbf{u}_1 + (\mathbf{v} \cdot \mathbf{u}_2)\,\mathbf{u}_2$.

      i. Hint: Since $\mathbf{u}_1$and $\mathbf{u}_2$ are orthogonal unit vectors, $\mathbf{v} \cdot \mathbf{u_1} = (a\mathbf{u}_1 + b\mathbf{u}_2) \cdot \mathbf{u}_1 = a\,(\mathbf{u}_1 \cdot \mathbf{u}_1) + b\,(\mathbf{u}_2 \cdot \mathbf{u}_1) = a\,(1) + b\,(0) = a$

7. [8, 7 points] If $\mathbf{a} = \langle 1, 4 \rangle$ and $\mathbf{b} = \langle 2, -3 \rangle$

   (a) Compute the vector projection, $\mathbf{c} = \text{proj}_{\mathbf{b}}\mathbf{a}$,of $\mathbf{a}$ onto $\mathbf{b}$.

      i. Hint: $\mathbf{c} = \text{proj}_{\mathbf{b}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{b}\|} \frac{\mathbf{b}}{\|\mathbf{b}\|}$

   (b) Show that $\mathbf{b}$ is orthogonal to $\mathbf{a} - \mathbf{c}$ by using the dot product.

      i. Hint: Use the $\mathbf{c}$ above to compute $\mathbf{a} - \mathbf{c}$ and then the dot product $(\mathbf{a} - \mathbf{c}) \cdot \mathbf{b}$