Mathematics 433 **Fall 2010** **Turn In Set: 7**

November 23, 2010

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"*To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty of nature. If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in*". -Richard Feynman (1918-1988)

Problems

1. Do both of the following.

   (a) Let $\phi : G \longrightarrow G'$ be an isomorphism of groups. Prove that the inverse function ϕ^{-1} is also an isomorphism.

   (b) Let $\phi : G \longrightarrow G'$ be an isomorphism of groups, let $x, y \in G$ and $\phi(x) = x'$.Prove that $\phi\left(x^{-1}\right) = \left(x'\right)^{-1}$

   (c) Give an example of two isomorphic groups such that there is more than one non-identity isomorphism between them.

2. Show that the functions $f = 1/x$ and $g = (x-1)/x$ generate a group of functions which is isomorphic to S_3, where the group operation is composition of functions.

3. Prove that the matrices $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ are conjugate elements in $GL_2(\mathbf{R})$ but are not conjugate in $SL_2(\mathbf{R})$.

4. Prove that the set $Aut(G)$ of all automorphisms of a group G forms a group under the operation of composition of functions. [An automorphism of a group G is an isomorphism with G as both the domain and the codomain.]

5. Determine the group of automorphisms of the following groups.

   (a) $\mathbf{Z}$

   (b) A cyclic group of order 10.

   (c) S_3