Designing Car Fenders

Drafts due: Thursday October 29, 2009

Paper due: Thursday November 5, 2009

____
Name

This exercise is a computational example of the use of **cubic splines** which have been heavily used in industrial design and computer graphics for decades. One of the first significant uses of cubic splines was in the design of the shape of the original Ford Mustang door. This is a simpler example and illustrates how one can join two cubic functions "smoothly" at the common point of their domains. Two of the many reasons for wanting such a smooth join are esthetic beauty and better resistance to external forces.

Writing Project 01

Directions: You may discuss this exercise with anyone but there is to be absolutely no collaboration when writing it up!

When designing the shape of a car fender, the following type of problem must be solved: Find two cubic polynomials p and q, with p defined on the interval $[1,3]$ and q defined on the interval $[3,9]$, and such that:

$$\begin{aligned} p(1) &= 3, \quad p(3) = 7, \quad q(3) = 7, \quad q(9) = -2 \\ p'(3) &= q'(3), \quad p''(3) = q''(3), \quad p''(1) = 0, \quad q''(9) = 0 \end{aligned}$$

The conditions $p(1) = 3$, $p(3) = 7$, $q(3) = 7$, and $q(9) = -2$ specify that the curved fender's profile will pass through the points $(1,3)$, $(3,7)$, and $(9,-2)$. The condition $p'(3) = q'(3)$ ensures the curve is "smooth" at the point where the graphs of p and q meet. The condition $p''(3) = q''(3)$ is also a smoothness condition and it also ensures that impact forces on the fender will be distributed in an efficient manner. The last conditions $p''(1) = 0$ and $q''(9) = 0$ are constraints that make it easier to manufacture the fender; they imply the fender will be "straight" at the ends. A curve that consists of cubic polynomials joined together in this "smooth" fashion (equal points, first derivatives and second derivatives at the join) are called **cubic splines.**

1. Let $p(x) = ax^3 + bx^2 + cx + d$ and $q(x) = \alpha x^3 + \beta x^2 + \gamma x + \delta$. Carefully explain how to find the values of $a, b, c, d, \alpha, \beta, \gamma$, and δ that satisfy the given conditions on p and q.

2. Use your calculator or a computer to draw the graphs of p and q on their respective intervals in the same window and then draw a careful copy on your paper. Be sure to note that the visual representation of both curves on $[1,9]$ appears to be both continuous and smooth.

3. Define a new function F by
$$F(x) = \begin{cases} p(x), & 1 \le x \le 3 \\ q(x), & 3 \le x \le 9 \end{cases}$$
and prove that F, F', and F'' are continuous on $[1,9]$.