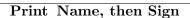
#### Proof M-2

# Accepted

# Not Accepted

I affirm this work abides by the university's Academic Honesty Policy.



- First due date Thursday, October 23.
- Turn in the the final version of your work on a separate sheet of paper with this page stapled in front.
- Do not include scratch work in your submission.
- Follow the Writing Guidelines of the Grading Rubric. (http://math.ups.edu/~bryans/Current/Fall\_2008/290inf\_Fall2008.html#tth\_sEc5.1)
- Retry: Only use material from the relevant section or earlier.
- Retry: Start over using a new sheet of paper.
- Retry: Restaple with new attempts first and this page on top.

"True eloquence consists in saying all that is necessary, and nothing but what is necessary." – La Rochefoucauld

## M-2 (Section FS)

**Definition:** If A is a square matrix then for each n > 1 we define  $A^n = A^{n-1}A$ . In addition we define  $A^1 = A$  and  $A^0 = I$  (the identity matrix).

- 1. Suppose A and B are square matrices of size n and that A is non-singular. Use mathematical induction to prove that  $(A^{-1}BA)^m = A^{-1}B^mA$  for every positive integer m.
- 2. Now suppose that B is also nonsingular and prove that the formula  $(A^{-1}BA)^m = A^{-1}B^mA$  holds for every integer (positive, negative and zero).
- 3. Use your formula and the matrices  $B = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$  and  $A = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$  and the vector  $\vec{x}_0 = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$  to compute  $B^m \vec{x}_0$ . What is the component by component limit of  $B^m \vec{x}_0$  as  $m \to \infty$ ?

## Notes:

- $A^{-1}BA$  should be a diagonal matrix.
- Recall the formula for powers of diagonal matrices (proven in class) and use it to compute  $B^m$ .