Proof M-2

Accepted **Not Accepted**

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

- First due date **Thursday, October** 23.
- Turn in the the final version of your work on a separate sheet of paper with this page stapled in front.
- Do not include scratch work in your submission.
- Follow the Writing Guidelines of the Grading Rubric. (http://math.ups.edu/~bryans/Current/Fall_2008/290inf_Fall2008.html#tth_sEc5.1)
- Retry: Only use material from the relevant section or earlier.
- Retry: Start over using a new sheet of paper.
- Retry: Restaple with new attempts first and this page on top.

"True eloquence consists in saying all that is necessary, and nothing but what is necessary." – La Rochefoucauld

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M-2 (Section FS)
Definition: If A is a square matrix then for each $n > 1$ we define $A^n = A^{n-1}A$. In addition we define $A^1 = A$ and $A^0 = I$ (the identity matrix).

1. Suppose A and B are square matrices of size n and that A is non-singular. Use mathematical induction to prove that $\left(A^{-1}BA\right)^m = A^{-1}B^mA$ for every positive integer m.
2. Now suppose that B is also nonsingular and prove that the formula $\left(A^{-1}BA\right)^m = A^{-1}B^mA$ holds for every integer (positive, negative and zero).
3. Use your formula and the matrices $B = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$ and $A = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$ and the vector $\vec{x}_0 = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$ to compute $B^m\vec{x}_0$. What is the component by component limit of $B^m\vec{x}_0$ as $m \to \infty$?

Notes:

- $A^{-1}BA$ should be a diagonal matrix.
- Recall the formula for powers of diagonal matrices (proven in class) and use it to compute B^m.
