Mathematics 433 Fall 2007 Problems to Turn in: 4

October 26, 2007

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"Mathematicians do not study objects, but relations among objects; they are indifferent to the replacement of objects by others as long as relations do not change. Matter is not important, only form interests them."
— Henri Poincaré

Problems

1. **You must do this problem.**
   (a) Prove the set $Aut\,(G)$of all automorphisms of a group G forms a group, the binary operation being the composition of functions.
   (b) Determine the group of automorphisms of each of the following groups.
      i. $(Z, +)$ (also known as Z^+)
      ii. A cyclic group of order 10.
      iii. S_3
2. Do **one** of the following.
   (a) Describe all homomorphisms $\phi : (Z, +) \to (Z, +)$. Determine which are one-to-one, which are onto and which are isomorphisms.
   (b) Do all of the following.
      i. Prove that if a group contains exactly one element of order 2, then that element is in the center of the group.
      ii. Suppose $\phi : G \to G'$ is an onto homomorphism. Prove, if G is cyclic, then G' is cyclic.
      iii. Suppose $\phi : G \to G'$ is an onto homomorphism. Prove, if G is abelian, then G' is abelian.
3. Do either of the following.
   (a) Find all subgroups of S_3 and determine which of these are normal.
   (b) Find all subgroups of the quaternion group and determine which of these are normal.
4. Let $\phi : G \to G'$ be an onto homomorphism and let N be a normal subgroup of G.
   (a) i. Show that the set $\phi\,(N) = \{\phi\,(n) : n \in N\}$ is a subgroup of G'.
      ii. Prove that $\phi\,(N)$ is a normal subgroup of G'.