Mathematics 433 Fall 2007 Problems to Turn in: 01

Due September 28

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"*Simple solutions seldom are. It takes a very unusual mind to undertake analysis of the obvious.*" — Alfred North Whitehead

Problems

When doing problems associated with matrices, you are not restricted to the material covered, so far, in our review.

1. You must do this problem. Do **two** of the following.

   (a) (Vandermonde Determinant)

   i. Prove that $\det \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix} = (b-a)(c-a)(c-b)$.

   ii. $(*)$ Prove an analogous formula for $n \times n$ matrices by using induction and row operations (in a clever fashion) to clear out the first column.

   (b) The Fibonacci sequence is defined by $f_1 = 1$, $f_2 = 1$, $f_{n+2} = f_n + f_{n+1}$ for all $n \geq 1$. Prove that for every positive integer n, $f_n < 2^n$.

   (c) Use induction to compute the determinant of $A = \begin{bmatrix} 2 & -1 & & & & & \\ -1 & 2 & -1 & & & & \\ & -1 & 2 & -1 & & & \\ & & & \ddots & & & \\ & & & -1 & 2 & -1 \\ & & & & -1 & 2 \end{bmatrix}$.

2. Do one of the following.

   (a) Prove that the Second Principle of Mathematical Induction implies the First Principle of Mathematical Induction.

   (b) Prove that the First Principle of Mathematical Induction implies the Second Principle of Mathematical Induction.

   (c) Let A, B be $m \times n$ and $n \times m$ matrices. Prove $I_m - AB$ is invertible if and only if $I_n - BA$ is invertible.

3. Let $P(A)$ be the set of all subsets of set A ($P(A)$ is called the **power set** of A.)

   (a) Prove that if A is a finite set with n elements then $P(A)$ has precisely 2^n elements.

   (b) Show that for any set (including infinite sets) A it is not the case that A is in one-one correspondence with $P(A)$. [Hint: the infinite case is much more interesting than the finite case. Let $f : A \to P(A)$ be any one-to-one function and show it cannot be onto by considering the subset of A consisting of all elements a that are not in their image, $f(a)$, under f .]