

Due October 8

Name

Be sure to re-read the **WRITING GUIDELINES rubric**, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**

“We used to think that if we knew one, we knew two, because one and one are two. We are finding that we must learn a great deal more about ‘and.’” (Sir Arthur Eddington)

Building Definite Integrals using Riemann Sums

There are two parts to this project. In the first you will use the formula for the surface area of spheres to obtain the formula for the volume inside a sphere of radius R . In the second you will do the reverse: use the formula for the volume inside spheres to obtain the formula for the surface area of a sphere of radius R . Thus, if you know either, calculus tells you the other.

1. Assume that you know the surface area of a 3-dimensional sphere, S^3 , of radius R is $A(R) = 4\pi R^2$ but pretend that you **do not know** the formula for the volume of that sphere.
 - (a) Use the fact that the volume inside S^3 can be filled by nested (empty) spheres of smaller radius, r to find a definite integral that represents the volume inside the sphere. Here, r ranges from 0 to R . Use the full Riemann Sum process and not just the three-step procedure.
 - (b) Evaluate your definite integral to obtain the standard formula for the volume inside a 3-dimensional sphere of radius R .
2. This time, assume that you know the formula for the volume inside a 3-dimensional sphere of radius R . Let $A(r)$ be the notation for the function that gives the surface area of a sphere of radius r but pretend that you **do not know** what the formula for $A(r)$ is.
 - (a) Go through our three-step process to build a definite integral, with integrand $A(r)$, that equals the volume inside the sphere of radius R , $\frac{4}{3}\pi R^3$.
 - (b) Explain how to use this equation to determine the formula for the surface area $A(r)$.