Mathematics 181 – **D** **Fall 2006**

Exam 5/Final

December 13, 2006

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Name

Technology used:______________________________ Directions:

- Be sure to include in-line citations every time you use technology.
- **Only write on one side of each page.**
- **When given a choice, specify which problem(s) you wish graded.**

Exam 5

Do any six (6) **of the following problems**

1. (15 points) The series $\frac{1}{2\cdot 3}-\frac{2}{3\cdot 4}+\frac{3}{4\cdot 5}-\frac{4}{5\cdot 6}+\frac{5}{6\cdot 7}-\frac{6}{7\cdot 8}+\cdots$ is an alternating series.
   (a) Write this series in sigma notation.
   (b) Determine if this series converges absolutely, converges conditionally or diverges. Include enough work to fully justify your answer.

2. (15 points) Given the series $\sum_{n=0}^{\infty}\frac{(x-3)^n}{n5^n}$. Find the Radius of Convergence and find all numbers x where the series converges absolutely and all numbers x where it converges conditionally.

3. (15 points) The power series $f(x)=\sum_{n=0}^{\infty}\frac{x^n}{n^2+1}$ converges absolutely for all numbers x in the interval $-1\le x\le 1$, does not converge conditionally for any number, and diverges for all numbers x with $|x|>1$.

   Determine the derivative series, $f\,'(x)$, and specify all values of x at which this derivative series converges absolutely and also all numbers x, if any, at which the series converges conditionally.

4. (15 points) Find the Taylor Polynomial, $P_3(x)$, of order 3 generated by $f(x)=(1+2x)^{-1/2}$ at $x=2$. [See "Useful Information" below for Taylor's Formula.]

5. (15 points) Write out the first five (5) terms of the binomial series for the function $f(x)=(1+2x)^{-1/2}$ and specify the numbers x for which this series converges. [This will be very different from the answer to the previous exercise.]

6. (15 points) Find a polynomial that will approximate $F(x)$ throughout the given interval with an error of magnitude less than 10^{-3}.

$$F(x)=\int_0^x \sin\left(t^2\right)\,dt,\qquad [0,1]$$

7. (15 points) The set of points satisfying the polar equation $r=4\cos(\theta)-9\sin(\theta)$ is a circle. Replace this polar equation with an equivalent Cartesian equation and specify the center and radius of this circle. You may not use a calculator on this problem.

Final Examination

You MUST answer the first three (3) problems

1. (10 points) Short answers:

   (a) Give an example of a convergent infinite series.

   (b) Give an example of a divergent infinite sequence.

   (c) Give an example of a nondecreasing, unbounded sequence

   (d) The Taylor Polynomial of order 6 generated by the function f at $x = 1$ is $P_6(x) = 12 - 4(x - 1) + 25(x-1)^2 - 12(x-1)^5 + 5(x-1)^6$. What is $f^{(5)}(1)$?

   (e) Give an example of a divergent improper integral.

2. (30 points) Evaluate three (3) of the following integrals.

   (a) $\int \frac{e^x \sin(e^x)}{\sqrt{1+\cos(e^x)}}\ dx$

   (b) $\int \frac{8x+8}{x(x^2+1)}\ dx$

   (c) $\int x^2 \sin(4x)\ dx$ Use Integration by Parts.

   (d) $\int (x+1/2)\tan\left(x^2+x\right)\sec^2\left(x^2+x\right)\ dx$

3. (15 points) Write out, in sigma notation, a Riemann Sum for the function $f(x) = x^2 + 1$ on the interval $[1, 3]$. Use the right hand endpoints of n subintervals of equal length to develop your answer. Do not take the limit of this sum, just write it out in sigma notation.

Do any three (3) of problems 4 – 7.

4. (15 points) Use an improper integral to find the volume of the solid obtained by rotating the unbounded region in the first quadrant between the x-axis and the graph of $y = e^{-x^2}$ about the y-axis.

5. (15 points) Do one (1) of the following:

   (a) The population of the world was estimated to be 3 billion in 1959 and 6 billion in 1999. What would an exponential model of population growth predict the population to be in 2007? [The actual population was 6,562,431,480 as of 21:06 GMT (EST+5) Dec. 10, 2006 – data from the US Census Bureau's population clock.]

   (b) The half-life of polonium 210 is about 138 days. Suppose a small amount of polonium dust was spilled in a restaurant. How many days would it take for the polonium to decay to 1% of the original amount?

6. (15 points) Do one (1) of the following:

   (a) Find the area of the surface generated by revolving the curve $y = \sqrt{x+1}$, $1 \le x \le 5$ about the x-axis.

   (b) Find the length of the curve $y = \frac{1}{3}\left(x^2+2\right)^{3/2}$ from $x = 0$ to $x = 3$.

7. (15 points) Use the error estimate for the Trapezoid Rule to determine a value of n that guarantees the trapezoidal estimate for the following integral is accurate to within 10^{-3}.

$$\int_{0.5}^{2.5} \cos(x^3)\ dx$$

Useful Information

- **Taylor's Formula**: If f has derivatives of all orders in an open interval I containing a, then for each positive integer n and for each x in I,

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n(x)$$

  where the remainder function $R_n(x)$ is given by $R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$