Mathematics 181 – D Fall 2006

Exam 4

November 14, 2006

_____
Name

Technology used: _____ Directions:

- Be sure to include in-line citations every time you use technology.
- Include a careful sketch of any graph obtained by technology in solving a problem.
- **Only write on one side of each page.**
- **When given a choice, specify which problem(s) you wish graded.**

Do any six (6) of the following problems

1. (15 points) Use the error bound formula for the Trapezoid Rule, $|E_T| \leq \frac{M(b-a)^3}{12n^2}$ to estimate the minimum number of subintervals needed to approximate the integral $\int_0^2 \sqrt{x+1}\, dx$ with an error of magnitude less than 10^{-4}.

2. (15 points) Does the following integral converge or diverge? Show all work.

   (a) $\int_{-\infty}^{\infty} \frac{x\ dx}{(x^2+9)^{3/2}}$

3. Below are six infinite series,

   (a) (2 points each) For **five** (5) of the six, state a reasonable test for checking for convergence or divergence and include a short sentence as to why that test is reasonable.

   (b) (10 points each) Choose **three** (3) of the series and determine if they converge or diverge. Show all details.

   i. $\sum_{n=1}^{\infty} \frac{6^n+1}{7^n}$
   ii. $\sum_{n=1}^{\infty} \frac{7^n}{6^n+1}$
   iii. $\sum_{n=1}^{\infty} \frac{[\ln(n)]^3}{n}$
   iv. $\sum_{n=2}^{\infty} \frac{6n^3-10n^2+1000n}{n^6-1}$
   v. $\sum_{n=1}^{\infty} \left(\frac{n-4}{n}\right)^n$
   vi. $\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$ [Be careful with the factorials.]

4. (15 points) Determine if the following series converges or diverges. If it converges, determine if the convergence is absolute or conditional.

   $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin(n)}{n^3}$

5. (10 points) Prove the following theorem.

   If $\sum_{n=1}^{\infty} a_n$ is a convergent series of nonnegative numbers, then $\sum_{n=1}^{\infty} \frac{a_n}{n}$ also converges.