Conceptual Review

5.1-5.4, 8.1 and bits of 8.2

Interval and Discrete Domain Analogies

$n^{\underline{p}} = \frac{n!}{p!} = n(n-1)(n-2)\cdots(n-p+1)$		
$D_n\left[n^{\underline{p}}\right] = pn^{\underline{p-1}}$		$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$
$D_n\left[n^{\underline{-p}}\right] = -p(n+1)^{\underline{-p-1}}$		$\frac{d}{dx}\left[x^{-n}\right] = -nx^{-n-1}$
$D_n\left[r^n\right] = (r-1)r^n$		$\frac{d}{dx}\left[a^x\right] = \ln(a)a^x$
$\sum n^{\underline{p}} = \frac{1}{p+1}n^{\underline{p+1}} + C$		$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$
$\sum n^{\underline{-p}} = \frac{1}{-p+1}(n-1)^{\underline{-p+1}} + C$, if $p \neq 1$		$\int x^{-n}dx = \frac{1}{-n+1}x^{-n+1} + C$, if $n \neq 1$
$\sum r^k = \frac{1}{r-1}r^k + C$, $r \neq 1$		$\int r^x dx = \frac{1}{\ln(r)}r^x + C$, $r \neq 1$
$\sum_{k=1}^{n} 1 = n$		$\int_a^b 1\,dx = b-a$
$\sum_{k=1}^{n} k = \frac{1}{2}n(n+1)$		$\int_a^b x\,dx = \frac{1}{2}\left(b^2-a^2\right)$
$\sum_{k=1}^{n} k^2 = \frac{1}{6}n(n+1)(2n+1)$		$\int_a^b x^2\,dx = \frac{1}{3}\left(b^3-a^3\right)$
$\sum_{k=1}^{n} k^3 = \frac{1}{4}n^2(n+1)^2$		$\int_a^b x^3\,dx = \frac{1}{4}\left(b^4-a^4\right)$
$D_n\left[\sum_{k=m}^{n-1} a(k)\right] = a(n)$	1st FT	$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$
If $D_n[A(n)] = a(n)$, $\sum_{k=m}^{n} a(k) = A(n+1) - A(m)$	2nd FT	If $\frac{d}{dx}[F(x)] = f(x)$ then $\int_a^b f(x)\,dx =$

Mixing Interval and Discrete Domain Functions, Part 1

Approximating areas under, average value of, or other properties of interval domain functions

- Start with a **continuous** function on an interval $[a,b]$.
- Partition the interval into n subintervals (which need not be the same size) using $P = \{a = x_0, x_1, \cdots, x_n = b\}$.
- Use notation: $[x_{k-1}, x_k]$ is the kth subinterval, Δx_k is the length of $[x_{k-1}, x_k]$, and $\|P\|$ is the length of the longest subinterval
- Select one point c_k in the k th subinterval for $k = 1, 2, \cdots, n$
- Form the sequence $a(k) = f(c_k)\,\Delta x_k$
- Form the finite sum (discrete antiderivative) $\sum_{k=1}^{n} f(c_k)\,\Delta x_k$
- Determine the limit $\lim_{\|P\|\to\infty} \sum_{k=1}^{n} f(c_k)\,\Delta x_k$ (By a theorem proven in advanced calculus, MATH 321, the limit is guaranteed to exist if f is continuous and the limit does not depend on which partitions P you use or on how you select points c_k in the subintervals).
- This limit gives an exact value, not an approximation, and is abbreviated with the notation $\int_a^b f(x)\,dx$.

Fundamental Theorem of Calculus

- Part 1 of the Fundamental Theorem of Calculus tells us that every continuous function is **guaranteed** to have an antiderivative. Specifically, $\int_a^x f(t)\, dt$ is an antiderivative of $f(x)$.
- Part 2 of the Fundamental Theorem of Calculus gives us a computational shortcut for computing the limit: $\lim_{\|P\|\to\infty} \sum_{k=1}^{n} f(c_k)\, \Delta x_k$. It requires that we know an antiderivative $F(x)$ of $f(x)$, but if we do, then $\int_a^b f(x) = \lim_{\|P\|\to\infty} \sum_{k=1}^{n} f(c_k)\, \Delta x_k = F(b) - F(a)$.