Mathematics 433 Fall 2006 Problems to Turn in: 9

November 17, 2006

First Turn In no later than November 29

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

"*Iron rusts from disuse; stagnant water loses its purity and in cold weather becomes frozen; even so does inaction sap the vigor of the mind.*" – Leonardo da Vinci

Problems

1. Do **one** of the following.
   (a) Let $G = D_4$ be the dihedral group of symmetries of the square.
      i. What is the stabilizer of a vertex? Of an edge?
      ii. G acts on the set of two elements consisting of the diagonal lines. What is the stabilizer of a diagonal?
   (b) Let $G = GL(n, R)$ operate on the set $S = R^n$ by left multiplication.
      i. Describe the decomposition of S into orbits for this operation.
      ii. What is the stabilizer of e_1?

2. Do **one** of the following.
   (a) Let G be a group and let H be the cyclic subgroup generated by an element x of G. Show that if left multiplication by x fixes every coset of H in G, then H is a normal subgroup of G.
   (b) A map $\phi : S \to S'$ of G - sets is called a **homomorphism** of G- sets if $\phi(gs) = g\phi(s)$ for all $s \in S$ and all $g \in G$. Let ϕ be such a homomorphism. Prove the following.
      i. The stabilizer $G_{\phi(s)}$ contains the stabilizer G_s.
      ii. The orbit of an element $s \in S$ maps onto the orbit of $\phi(s)$.

3. Let G be the group of rotational symmetries of a cube C. Two regular tetrahedra Δ and Δ' can be inscribed in C, each using half of the vertices. What is the order of the stabilizer of Δ?