Mathematics 433 **Fall 2006** **Problems to Turn in: 2**

September 15

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"The one real object of education is to have a man in the condition of continually asking questions."
-Bishop Mandell Creighton

Problems

1. Do **both** of the following.
   (a) Let a, b be elements of a group G. Show that the equation $ax = b$ has a unique solution in G.
   (b) Let G be a group, with multiplicative notation. Define an **opposite group** G° with law of composition $a \circ b$ as follows: The underlying set is the same as for G, but the law of composition is the opposite; that is, define $a \circ b = ba$. Prove that this defines a group.

2. Do both of the following:
   (a) Prove that if G is a group with the property that the square of every element is the identity, then G is abelian.
   (b) Let G be a finite group. Show that the number of elements x of G such that $x^3 = e$ is odd. Show that the number of elements x of G for which $x^2 \neq e$ is even.

3. Do any two of the following
   (a) Prove that every subgroup of a cyclic group is cyclic.
   (b) Prove that the set of elements of finite order in an abelian group is a subgroup.
   (c) If H and K are subgroups of a group G, show that $H \cap K$ is a subgroup of G. Adapt your proof to show that the intersection of any number of subgroups of G, finite or infinite, is again a subgroup of G. Notational hint: Let C be a collection of subgroups of G. Then we can denote the intersection of all the subgroups in C by
   $$\bigcap_{H \in C} H$$
   .

4. Show by example that the product of elements of finite order in a nonabelian group need not have finite order.