Mathematics 433 **Fall 2006** **Problems to Turn in:** 01

August 29, 2006

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"*Simple solutions seldom are. It takes a very unusual mind to undertake analysis of the obvious.*" — Alfred North Whitehead

Problems

When doing problems associated with matrices, you are not restricted to the material covered, so far, in our review.

1. You must do this problem. Do **two** of the following.

   (a) (Vandermonde Determinant)

   i. Prove that $\det \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix} = (b-a)(c-a)(c-b)$.

   ii. ($*$) Prove an analogous formula for $n \times n$ matrices by using induction and row operations (in a clever fashion) to clear out the first column.

   (b) Find a formula for $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^n$, and prove it by induction.

   (c) Use induction to compute the determinant of $A = \begin{bmatrix} 2 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & -1 & 2 & -1 & & \\ & & & \ddots & & \\ & & & -1 & 2 & -1 \\ & & & & -1 & 2 \end{bmatrix}$.

   (d) Use Mathematical Induction to prove that if $A_1, \cdots, A_n$ are square, invertible, $m \times m$ matrices then the product $A_1 \cdots A_n$ is also invertible and

   $$(A_1 \cdots A_n)^{-1} = A_n^{-1} \cdots A_1^{-1}.$$

2. Do one of the following.

   (a) Prove that the Second Principle of Mathematical Induction implies the First Principle of Mathematical Induction.

   (b) ($*$) Let A, B be $m \times n$ and $n \times m$ matrices. Prove $I_m - AB$ is invertible if and only if $I_n - BA$ is invertible.

3. Let $P(A)$ be the set of all subsets of set A ($P(A)$ is called the **power set** of A.) Show that for any set (including infinite sets) A it is not the case that A is in one-one correspondence with $P(A)$. [Hint: the infinite case is much more interesting than the finite case. Let $f : A \to P(A)$ be any one-to-one function and show it cannot be onto by considering the subset of A consisting of all elements a that are not in their image, $f(a)$, under f.]