Math 290 Fall 2006 Project 10

Due November 10

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Name

Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**

"The road to wisdom? Well its plain and simple to express: Err and err and err again, but less and less and less." -Piet Hein, poet and scientist (1905-1996)

Project Problem

Do **both** of the following.

1. Find the determinant of the $n \times n$ matrix A below. To be specific, the ijth entry of A is

$$[A]_{ij} = \begin{cases} 1 \text{ if } i+j = n+1 \\ 0 \text{ otherwise} \end{cases}$$

$$\begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

   Give your answer in terms of the remainder you get when you divide n by 4.

2. Recall that a matrix A is **skew-symmetric** if $A^t = -A$.

   (a) Give an example of a non-zero, 2×2 matrix.

   (b) **Prove** that the diagonal elements of an $n \times n$ skew-symmetric matrix are zero.

   (c) Consider an $n \times n$ skew-symmetric matrix A, where n is an odd integer. Prove that A is singular.