Not to be turned in

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Name

Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**
" 'Know thyself?' If I knew myself, I'd run away." – Johann von Goethe

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Do two (2) of the following.

1. If it is possible, how should the coefficients a, b, and c be chosen so the system below has solution set $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \right\}$?

$$\begin{aligned} ax + by + cz &= 3 \\ ax - y + cz &= 1 \\ x + by - cz &= 2 \end{aligned}$$

2. Suppose A is an $m \times n$ matrix and I_m is the $m \times m$ identity matrix. Let E be the matrix obtained by performing a single elementary row operation on I_m. We call any such matrix E an elementary matrix of size m and we use (EA) to denote the matrix obtained by performing the same elementary row operation that is encoded by E on matrix A. Prove that if A is a nonsingular $m \times m$ matrix, then there is a sequence of elementary matrices $E_1, E_2, \cdots, E_n$ for which $A = (E_n \cdots (E_3 (E_2 (E_1 I_m))) \cdots)$.

3. Consider the 2×2 system of equations

$$\begin{aligned} ax_1 + bx_2 &= f_1 \\ cx_1 + dx_2 &= f_2. \end{aligned}$$

   (a) Show that if $ad - bc \neq 0$, then this system is consistent and has exactly one solution. [Hint: be careful. You can't divide by a unless you know it is not zero.]

   (b) Show that if $ad - bc = 0$,this system can not have exactly one solution. [Hint: try to remove a variable using row operations that do not include division.]