Mathematics 122A,B Fall 2004 Project 4

Due September 28

Study Group Members **Name**

Only write on one side of each page.

I encourage you to work with others on this quiz. As with all writing you should work out the details in a draft before writing a final solution. Be sure to follow the writing guidelines listed in the course information sheet unless explicitly directed to do otherwise in the problem statement. You do not need to include every algebra or arithmetic step but you should include enough detail to allow a member of your target audience to reconstruct any missing steps. Be sure to include in-line citations, with page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. If you include graphs, they should be done carefully on graph paper. Finally, there is to be no collaboration in the writing of your solution even if you worked out the details with other people.

"Mathematics is the language with which God has written the universe" -Galileo Galilei, physicist and astronomer (1564-1642)

Problems

This problem is designed to develop a "better" approximation formula than those given by the Trapezoidal or Midpoint Rules.

Choose a specific interval $[a, b]$ and non-trivial linear, quadratic, cubic, and quartic functions. Call the functions $f_1(x)$, $f_2(x)$, $f_3(x)$, and $f_4(x)$, respectively.

1. For each of these functions compute the following:

   (a) The exact value of
   $$I = \int_a^b f_k(x)\ dx.$$
   (Use the First Fundamental Theorem of Calculus.)

   (b) The Midpoint approximations of I, M_n, using $n = 100$ and $n = 1000$. Include the formulas you input into your calculator or spreadsheet.

   (c) The Trapezoid approximations of I, T_n, using $n = 100$ and $n = 1000$. Include the formulas you input into your calculator or spreadsheet.

2. Use the data from part 1. to determine a weighted average of M_n and T_n that yields, for this data, the best possible approximation of I. Explain why your data suggest this approximation.

3. Use an appropriate graph to give a geometric argument that for large values of n and for "nice" functions f, your approximation will be an improvement on both the Midpoint and Trapezoidal Rules.

4. Write out, and algebraically justify, a formula for your improved approximation using the same format as in the "Trapezoidal Rule" box on page 335 of our text.