Mathematics 221 Fall Project 5

Due October 21

Name

Directions: Be sure to follow the guidelines for writing up projects as specified in the course information sheet (passed out on the first day of class). Whenever appropriate, use in-line citations, including page numbers and people consulted when you present information obtained from discussion, a text, notes, or technology. **Only write on one side of each page.**

"I never know how much of what I say is true." — Bette Midler

0.1 Project Description

Do one (1) of the following. All problems are "computational" but the last one requires more attention to details.

1. Do both of the following:

   (a) Assume that all functions are differentiable. If $z = f(x, y)$, where $x = r\cos\theta$, and $y = \sin\theta$, find
   $$\frac{\partial z}{\partial r}, \quad \frac{\partial z}{\partial \theta} \quad \text{and} \quad \frac{\partial^2 z}{\partial r \partial \theta}.$$

   (b) Assume that all functions are differentiable. Show that any function of the form
   $$z = f(x + at) + g(x - at)$$
   is a solution of the *wave equation*
   $$\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}$$

2. Suppose $z = f(x, y)$ is a function on two intermediate variables and $x = x(s, t)$, $y = y(s, t)$ are functions on two independent variables s, t. Derive the formula for
   $$\frac{\partial^2 z}{\partial t\, \partial s}.$$
   [Recall that $\frac{\partial^2 z}{\partial t\, \partial s} = f_{st}(x, y)$.]