Mathematics 221 Fall 1998 Exam 2

October 6, 1998

Technology used: ______________________________

Textbook/Notes used: ______________________________

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. **Only write on one side of each page.**

The Problems

1. Write equations, in parametric form, of the line of intersection of the planes $x - 2y + 4z = 2$ and $x + y - 2z = 5$.

2. Find an equation for the plane consisting of all the points equidistant from the points $(1, 1, 1)$ and $(1, 2, 3)$ ..

3. Find an equation of the plane that passes through the line of intersection of the planes $x - z = 1$ and $y + 2z = 3$ and is perpendicular to the plane $x + y - 2z = 1$.

4. Find parametric equations for the line in the plane $z = 3$ that makes a $30°$ angle with $\mathbf{i}$ and a $60°$ angle with $\mathbf{j}$..

5. Find the point(s) of intersection of the paraboloid $y^2 + z^2 = 2x$ and the curve traced out by the position function $\overrightarrow{r}(t) =< e^t, e^t \cos(t), e^t \sin(t) >$, $0 \le t \le \infty$.

6. Find the length of the space curve
$$\overrightarrow{r}(t) =< 6t^3, -2t^3, -3t^3 >, \quad \text{from } t = -1 \text{ to } t = 1.$$

7. Find the unit tangent, normal and binormal vectors $\mathbf{T}, \mathbf{N}$ and $\mathbf{B}$ for the space curve $\overrightarrow{r}(t) = (t^3/3)\ \mathbf{i} + (t^2/2)\ \mathbf{j}$, $t > 0$.

8. Find the curvature κ for $\overrightarrow{r}(t) =< \cos^3(t), \sin^3(t), 0 >$, $0 < t < \pi/2$.

9. Sketch the region bounded by the surfaces $z = \sqrt{x^2 + y^2}$, and $x^2 + y^2 = 1$ for $1 \le z \le 2$.

10. Prove that if $\overrightarrow{u}(t)$ is a differentiable function with outputs in R^3 and $f(t)$ is a differentiable scalar function, then
$$\frac{d}{dt}\left[\overrightarrow{u}(f(t))\right] = f'(t)\, \overrightarrow{u}'(f(t)).$$

11. Find the length of the ellipse
$$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$
[Hint: parametrize the ellipse using vectors of the form $< a\cos(t), b\sin(t) >$.]

12. The distance between two lines in R^3 that do not intersect is defined to be the length of a line segment PQ where P is on one line, Q is on the other line and segment PQ is perpendicular to both lines. Find the distance between the skew lines given by

$$\begin{aligned} x &= 1+t,\ y=1+6t,\ z=2t \text{ and} \\ x &= 1+2s,\ y=5+15s,\ z=-2+6s. \end{aligned}$$

13. Show that if a particle's speed is constant (i.e., $|\overrightarrow{v}(t)| = c$) then its acceleration is either zero or normal to its path.