Mathematics 122A,B Fall 2002 Quiz 1

August 31, 2001

Study Group Members **Name**

Only write on one side of each page.

I encourage you to work with others in the class on this quiz. As with all writing you should work out the details in a draft before writing a final solution. Be sure to follow the 5 basic guidelines listed in the course information sheet unless explicitly directed to do otherwise in the problem statement. You do not need to include every algebra or arithmetic step but you should include enough detail to allow a member of your target audience to reconstruct any missing steps. Be sure to include in-line citations, with page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. If you include graphs, they should be done carefully on graph paper. Finally, there is to be no collaboration in the writing of your solution even if you worked out the details with other people.
"Obvious" is the most dangerous word in mathematics." – Eric Temple Bell

Problems

1. The first few terms of a certain sequence are given below. Find a formula for the k'th term of the sequence. Your answer should have the form: $a(k) =$ "Your formula here", $k = 0, 1, 2, 3, \cdots$.

$$5, 3, 9, 59, 213, 555, 1193, 2259, 3909, 6323, 9705, \cdots.$$

2. In class we defined the following sequences: $a_0(k) = k^0$, $a_1(k) = k^1$, $a_2(k) = k(k-1)$, $a_3(k) = k(k-1)(k-2)$, $k = 0, 1, 2, 3, 4, 5, \cdots$ and noted

$$\begin{aligned} D_k(a_1) &= 1a_0 \\ D_k(a_2) &= 2a_1 \\ D_k(a_3) &= 3a_2. \end{aligned}$$

   (a) Find a sequence $a_4(k)$, $k = 0, 1, 2, 3, \cdots$ that satisfies

$$D_k(a_4) = 4a_3.$$

   (b) State a formula for sequences $a_n(k)$ that works for each of the special cases a_1, a_2, a_3, a_4 and prove that your formula satisfies the property that the discrete derivative of $a_n(k)$ is the sequence $na_{n-1}(k)$.That is, using the letter n, show

$$D_k(a_n) = na_{n-1}.$$