1 Mathematics 433 Fall 2000 Problems to Turn in: 3

September 7, 2000

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
" *You don't understand anything until you learn it more than one way.*" – Marvin Minsky

Problems

1. You must do this problem.

   (a) If H is a subgroup of G, then by the **centralizer** $C(H)$ of H we mean the set $\{x \in G : xh = hx$ for all $h \in$
   Prove that $C(H)$ is a subgroup of G.

   (b) Must the centralizer of an element of a group be Abelian?

   (c) Must the center of a group be Abelian?

2. Do one (1) of the following.

   (a) Suppose that G is a group of order 16 and that, by direct computation, you know that G has at least nine elements x such that $x^8 = e$.

      i. Can you conclude that G is not cyclic?
      ii. What if G has at least five elements x such that $x^4 = e$?
      iii. Generalize your results as a reasonable conjecture.

   (b) If G is an Abelian group and contains cyclic subgroups of orders 4 and 5, what other sizes of cyclic subgroups must G contain?