Mathematics 433 Fall 2000 Problems to Turn in

August 29, 2000

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"*Simple solutions seldom are. It takes a very unusual mind to undertake analysis of the obvious.*" — Alfred North Whitehead

Problems

When doing problems associated with matrices, you are not restricted to the material covered, so far, in our review.

1. Do one of the following.

   (a) Prove that the First Principle of Mathematical Induction implies the Second Principle of Mathematical Induction.

   i. [Note: This is **not** an induction proof. It is a proof **about** mathematical induction. The result completes the claim in the textbook that the two principles of Mathematical Induction are each equivalent to the Well Ordoring Principle via

   $$\mathbf{WOP} \Longrightarrow^{\text{in text}} \text{1st} \Longrightarrow \text{2nd} \Longrightarrow^{\text{ask in class}} \mathbf{WOP}$$

   since:

   (b) (∗) Let A, B be $m \times n$ and $n \times m$ matrices. Prove $I_m - AB$ is invertible if and only if $I_n - BA$ is invertible.

2. Do one of the following.

   (a) (Vandermonde Determinant)

   i. Prove that $\det \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix} = (b-a)(c-a)(c-b)$.

   ii. (∗) Prove an analogous formula for $n \times n$ matrices by using induction and row operations (in a clever fashion) to clear out the first column.

   (b) Find a formula for $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^n$, and prove it by induction.

   (c) Use induction to compute the determinant of $A = \begin{bmatrix} 2 & -1 & & & & \\ & -1 & 2 & -1 & & \\ & & -1 & 2 & -1 & \\ & & & \ddots & & \\ & & & -1 & 2 & -1 \\ & & & & -1 & 2 \end{bmatrix}$.

3. Do **both** of the following.

   (a) Let a, b be elements of a group G. Show that the equation $ax = b$ has a unique solution in G.

(b) Let G be a group, with multiplicative notation. Define an **opposite group** G^0 with law of composition $a \circ b$ as follows: The underlying set is the same as for G, but the law of composition is the opposite; that is, define $a \circ b = ba$. Prove that this defines a group.