INDEPENDENCE AND BAYESIAN NETWORKS

Today

- How can independence help?
- Bayesian networks

Inference by Enumeration

- n random variables
- d domain size
- Worst-case time is $O(d^n)$
- Space is $O(d^n)$ to save entire table in memory

- Is there something better?

The Chain Rule

Bayes Rule

Independence of Random Variables

Conditional Independence

Space Complexity

Moving away from numerical quantities

"The traditional definition of independence uses equality of numerical quantities, as in

$$p(x, y) = p(x)p(y)$$

suggesting that one must test whether the joint distribution of X and Y is equal to the product of their marginals in order to determine whether X and Y are independent. By contrast people can easily and confidently detect dependencies, even though they may not be able to provide precise numerical estimates of probabilities. A person who is reluctant to estimate the probability of being burglarized the next day or of having a nuclear war within five years can nevertheless state with ease whether the two events are dependent, namely, whether knowing the truth of one proposition will alter the belief of the other."

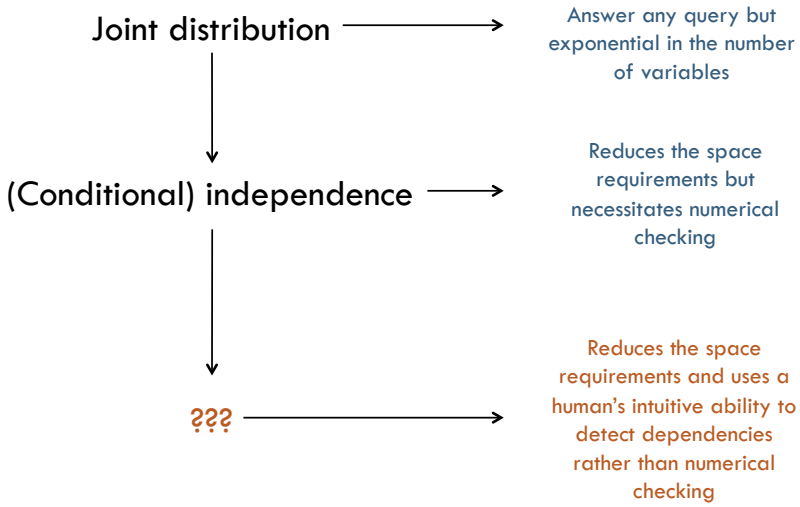
- Judea Pearl

Moving away from numerical quantities

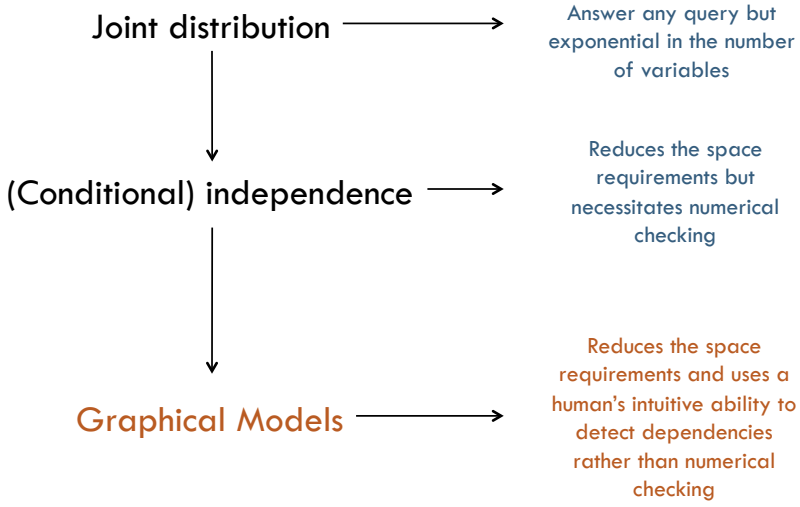
"It is usually easy for a domain expert to decide what direct influences exist in the domain – much easier, in fact, than actually specifying the probabilities themselves"

- Humans can "easily and confidently" detect dependencies
- Move away from numerical representation of the joint distribution (or the conditional distributions) to a representation that encodes dependencies

Probabilistic Inference



Probabilistic Inference



Bayesian Networks

- A Bayesian Network is a directed acyclic graph that represents the conditional independencies of a set of random variables
  - Each r.v. corresponds to a node
  - A directed edge represents a direct influence
  - The conditional distribution of each node given its parents must be explicitly specified

Bayesian Network Examples

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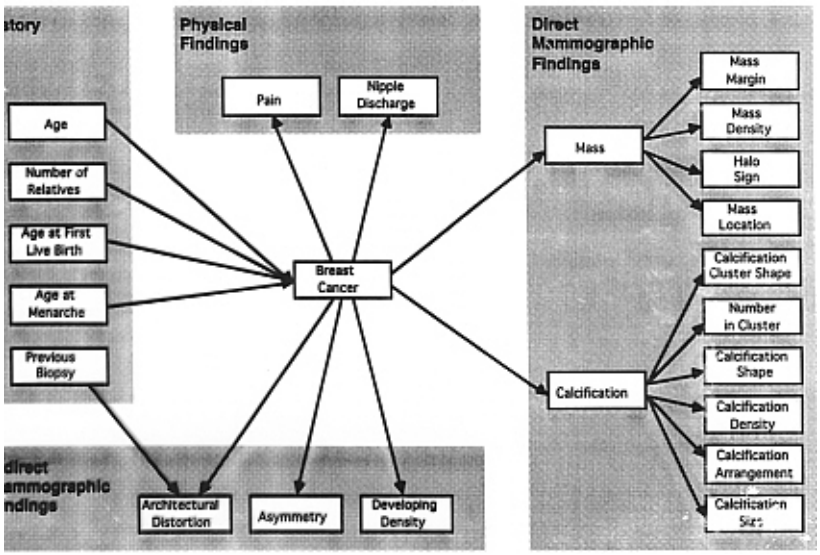
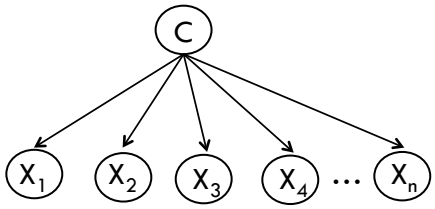
Example: MammoNet



Commonly used Bayesian Networks

- Naïve (Idiot) Bayes Classifier
  - Commonly used for text classification
  - C is the class (topic or label) of the document
  - The X variables represent the words in the document



Today an earthquake occurred in Southern California…

Representing the joint using a BN

- Given a BN over a set of random variables, the joint distribution can be factor as

$$p(x_1, \ldots, x_N) = \prod_{i=1}^{N} p(x_i | \text{parents}(x_i))$$