

BAYESIAN NETWORKS

Today

- Definition of Bayesian networks
- Reading off independencies
 - Types of Connection
 - D-separation algorithm

Bayesian Networks

- A **Bayesian Network** is a directed acyclic graph that represents the conditional independencies of a set of random variables
 - Each random variable corresponds to a node
 - A directed edge represents a direct influence
 - The conditional distribution of each node given its parents must be explicitly specified

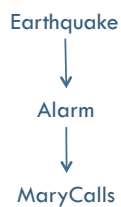
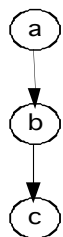
Representing the joint using a BN

- Given a BN over a set of random variables, the joint distribution can be factor as

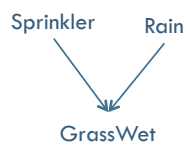
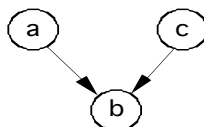
$$p(x_1, \dots, x_N) = \prod_{i=1}^N p(x_i | \text{parents}(x_i))$$

Three Types of Connections

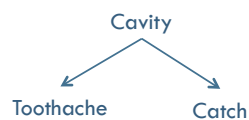
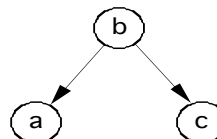
Linear



Converging



Diverging



Connection patterns and independence

- **Linear connection:**

- **Converging connection:**

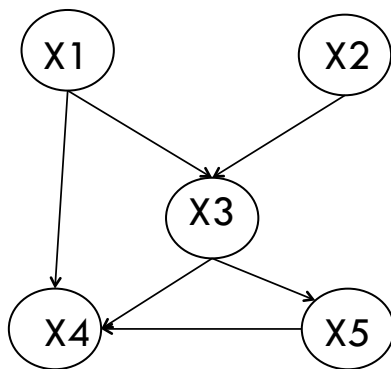
- **Divergent connection:**

D-Separation

- Algorithm to determine independencies in BN
- Query: Are two variables X_i and X_j independent?
- Check all paths between X_i and X_j
 - ▣ If all paths are blocked, then independent
 - ▣ If any path is not blocked then not independent

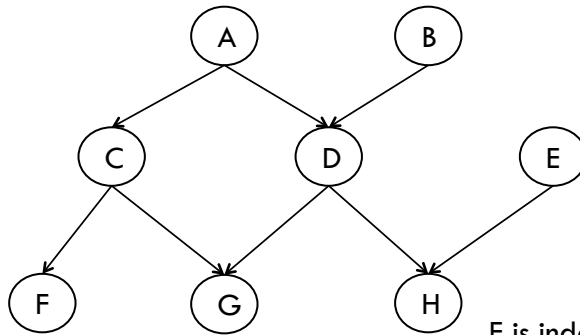
A path is blocked if for any connection on the path the two end variables are independent

List the independencies in the following Bayesian Network



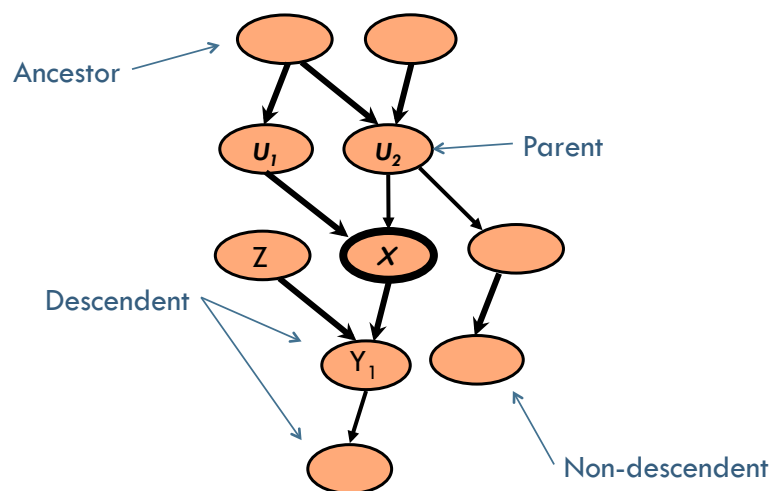
- X2 is independent of X3?
- X2 is independent of X4?
- X2 is independent of X4 given X3?
- What would make X2 and X4 ind.?

List the independencies in the following Bayesian Network



E is independent of G?
 C is independent of D?
 C is independent of D given G?
 F is independent of A given {C,D}?

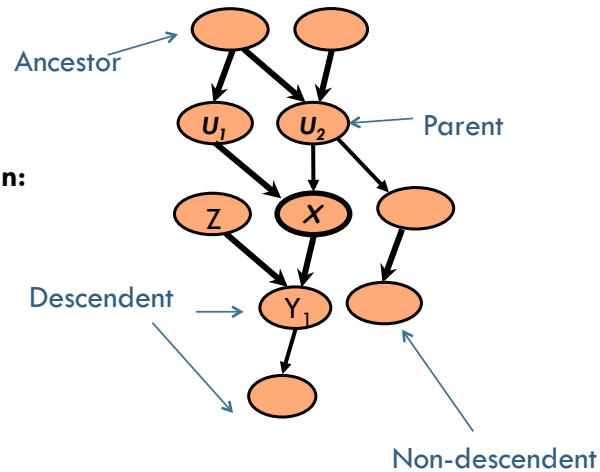
Bayesian Networks terminology



Independence assumptions encoded in the Bayesian Network

Local Markov Assumption:

A node X is independent of its non-descendants given its parents



Independence assumptions encoded in the Bayesian Network

Markov Blanket:

A node X is conditionally independent of all other nodes given its parents, children, and children's parents

