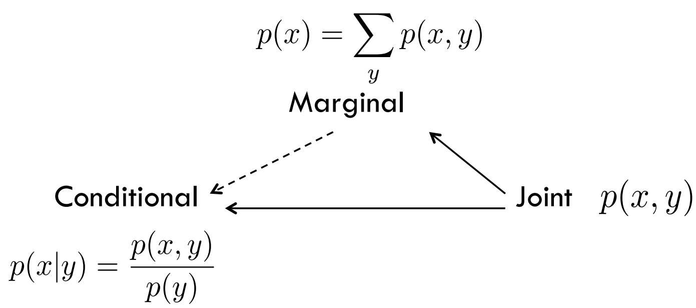
INDEPENDENCE AND BAYESIAN NETWORKS

Today

- Reading
  - AIMA Ch. 13, Ch. 14.1-14.4
- Goals
  - Build up to notions of independence
  - Bayesian networks

Summary of distributions so far



Probabilistic Inference

Probabilistic inference refers to the task of computing some desired probability given other known probabilities (evidence)

Inference by Enumeration

- Have a set of random variables $\{X_1, X_2, \ldots, X_n\}$
- Partition the set of random variables into:
  - Evidence variables: $E_1=e_1, E_2=e_2, \ldots E_k=e_k$
  - Query variables: Q
  - Hidden (misc.) variables: $H_1, H_2, \ldots, H_r$

We're interested in computing: $p(Q \mid E_1=e_1, E_2=e_2, \ldots, E_k=e_k)$

Step One: select the entries in the table consistent with the evidence (this becomes our world)

Step Two: sum over the H variables to get the joint distribution of the query and evidence variables

Step Three: Normalize

$$p(Q, e_1, \ldots, e_k) = \sum_{(h_1, \ldots, h_r)} p(Q, e_1, \ldots, e_k, h_1, \ldots, h_r) \longrightarrow p(Q|e_1, \ldots, e_k) = \frac{1}{Z} \cdot p(Q, e_1, \ldots, e_k)$$

$$Z = \sum_q p(Q = q, e_1, \ldots, e_k)$$

Example of using Bayes' Rule

- Provides a "causal" way of assessing probabilities

$$p(\text{disease}|\text{symptom}) = \frac{p(\text{symptom}|\text{disease}) \cdot p(\text{disease})}{p(\text{symptom})}$$

t = temperature
f = flu

Compute $p(+f \mid +t)$

Moving away from numerical quantities

"The traditional definition of independence uses equality of numerical quantities, as in

$$p(x, y) = p(x)p(y)$$

suggesting that one must test whether the joint distribution of X and Y is equal to the product of their marginals in order to determine whether X and Y are independent. By contrast people can easily and confidently detect dependencies, even though they may not be able to provide precise numerical estimates of probabilities. A person who is reluctant to estimate the probability of being burglarized the next day or of having a nuclear war within five years can nevertheless state with ease whether the two events are dependent, namely, whether knowing the truth of one proposition will alter the belief of the other."

- Judea Pearl