

# PROBABILISTIC REASONING OVER TIME

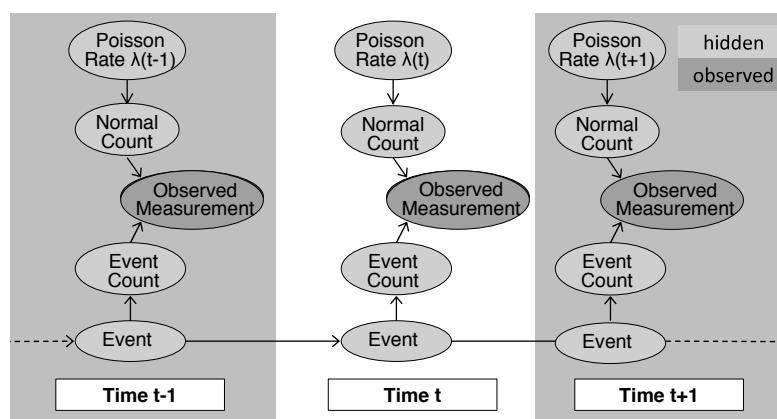
## Today

- Reading
  - ▣ AIMA Chapter 15.1-15.2, 15.5
  
- Goals
  - ▣ Particle Filters
  - ▣ In-class inference practice

## Modeling uncertainty over time

- Sometimes, we want to model a *dynamic* process: the value of the random variables change over time
  - Price of a stock
  - Patient stats, e.g. blood pressure, heart rate, blood sugar levels
  - Traffic on California highways
  - Pollution, humidity, temperature, rain fall, storms
  - Sensor tracking and detection

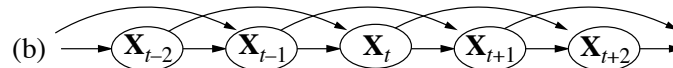
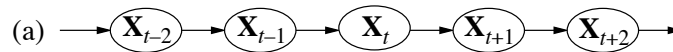
## Examples of DBN



J. Hutchins et al. Probabilistic analysis of a large-scale urban traffic sensor data set.

## Transition Model

- The transition model specifies the probability of  $X_t$  given the history  $X_{0:t-1}$
- **Markov Assumption: the state variable  $X_t$  depends on a finite and fixed subset of  $X_{0:t-1}$** 
  - ▣ First order Markov Process:  $P(X_t | X_{0:t-1}) = P(X_t | X_{t-1})$
  - ▣ Second order Markov Process:  $P(X_t | X_{0:t-1}) = P(X_t | X_{t-1}, X_{t-2})$



## Transition Model

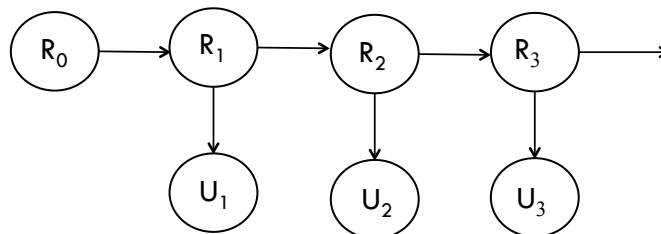
- **Stationarity Assumption: the conditional distribution  $P(X_t | X_{t-1})$  is the same for all  $t$**
- Only need to specify one conditional distribution for all edges

## Sensor (emission) model

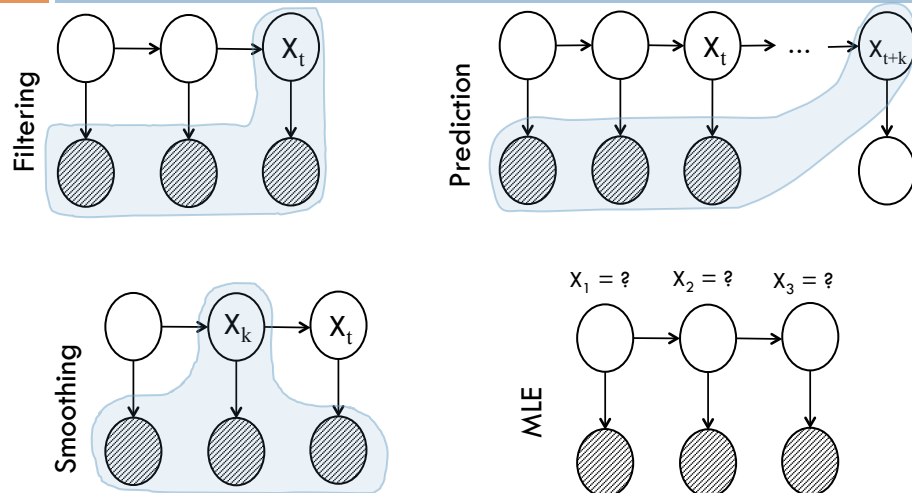
- The sensor model specifies the probability of the evidence  $E_t$  given the states  $X_{0:t}$  and the evidence  $E_{0:t-1}$
- **Sensor Markov Assumption: the evidence  $E_t$  is independent of every other random variable given  $X_t$** 
  - ▣ The state encompasses all relevant information for generating the evidence

## Hidden Markov Model

- A **Hidden Markov Model** is the simplest type of DBN
  - ▣ State is represented by a single discrete random variable



## Inference Tasks



## Approximate Inference in Dynamic BN

- Recall approximate inference algorithms from previous lecture
  - ▣ Direct sampling, rejection sampling, likelihood weighting
  - ▣ Gibbs sampling
- Likelihood weighting applied to DBN (with some modifications) is known as a **Particle filter**

## Particle Filtering

- Likelihood weighting fixes the evidence variables and samples only the non-evidence variables
- Introduces a weight to correct for the fact that we're sampling from the prior distribution instead of the posterior distribution

$$\text{weight} = p(e_1 | \text{Parents}(e_1)) * p(e_2 | \text{Parents}(e_2)) \dots$$

## Particle Filtering

- **Initialize**
  - ▣ Draw  $S$  particles (i.e. samples) for  $X_0$  from the prior distribution  $p(X_0)$
- **Propagate**
  - ▣ Propagate each particle forward by sampling a value  $x_{t+1}$  from  $p(X_{t+1} | X_t)$
- **Weight**
  - ▣ Weight each particle by  $p(e_{t+1} | X_{t+1}=x_{t+1})$
- **Resample**
  - ▣ Generate  $S$  new particles by sampling proportional to the weights. The new particles are unweighted

# Particle Filtering

- Particles: track samples of states rather than an explicit distribution

