BAYESIAN NETWORKS

Today

- Reading
  - AIMA Ch. 14.1-14.4 (Bayesian Networks)
- Goals
  - Definition of Bayesian networks
  - Examples
  - D-separation algorithm

More Practice…

Cavity	Catch	ToothAche	P
T	T	T	0.16
T	T	F	0.08
T	F	T	0.01
T	F	F	0.01
F	T	T	0.02
F	T	F	0.16
F	F	T	0.06
F	F	F	0.5

Queries:
p(Catch | Toothache = false)
~~p(Toothache | Cavity = true)~~
p(Cavity, Catch)
p(Cavity | Toothache = false, Catch = true)

Probabilistic Inference

Joint distribution → Answer any query but exponential in the number of variables

↓

(Conditional) independence → Reduces the space requirements but necessitates numerical checking

↓

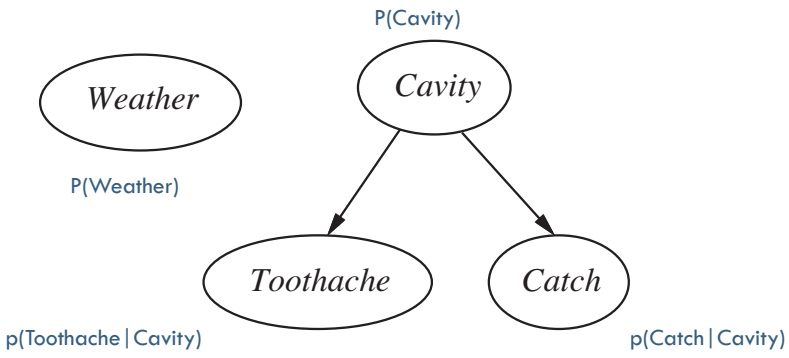
Graphical Models → Reduces the space requirements and uses a human's intuitive ability to detect dependencies rather than numerical checking

Bayesian Networks

- A Bayesian Network is a directed acyclic graph that represents the conditional independencies of a set of random variables
  - Each random variable corresponds to a node
  - A directed edge represents a direct influence
  - The conditional distribution of each node given its parents must be explicitly specified

Bayesian Network Examples

- Weather = {rainy, sunny, cloudy, snowy}
- Cavity = { true, false}
- Toothache = {true, false}
- Catch = {true, false}

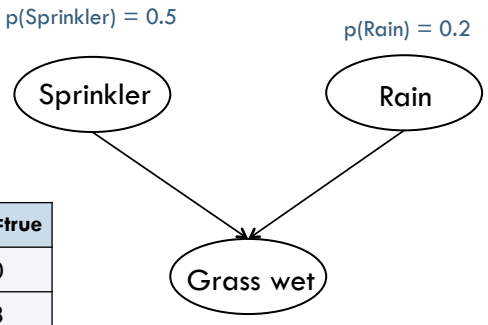


Representing the joint using a BN

- Given a BN over a set of random variables, the joint distribution can be factor as

$$p(x_1, \ldots, x_N) = \prod_{i=1}^{N} p(x_i | \text{parents}(x_i))$$

Bayesian Network Examples



p(Grass | Sprinkler, Rain)

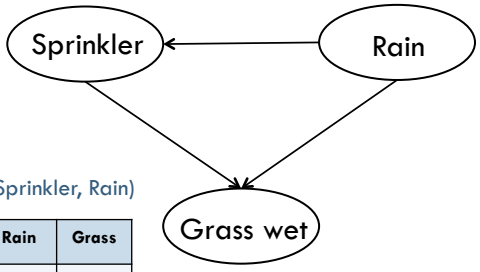
Sprink	Rain	Grass=true
F	F	0.0
F	T	0.8
T	F	0.9
T	T	0.99

Bayesian Network Examples

p(Sprinkler | Rain)

Rain	Sprink
F	0.5
T	0.01

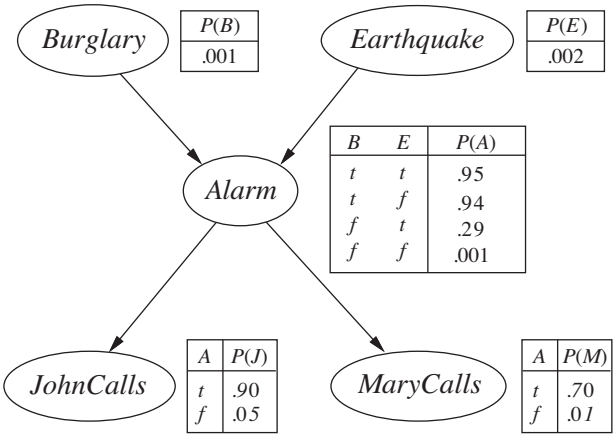
p(Rain) = 0.2



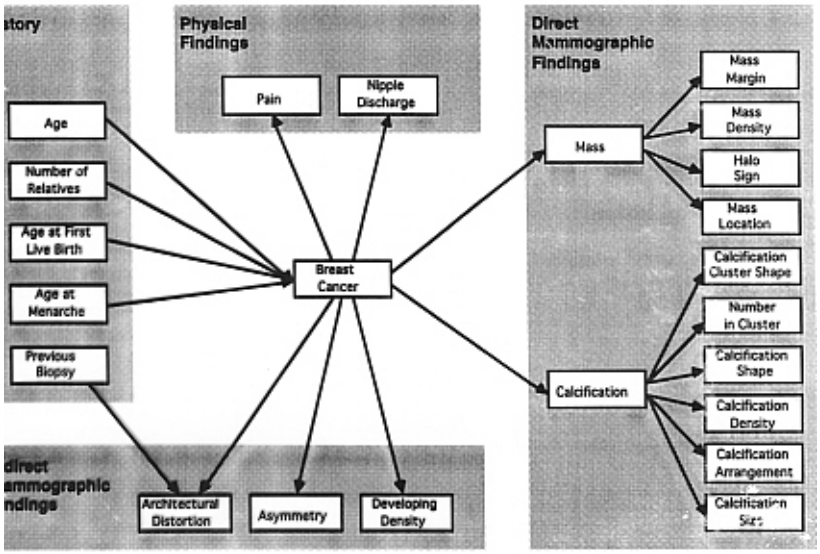
p(Grass | Sprinkler, Rain)

Sprink	Rain	Grass
F	F	0.0
F	T	0.8
T	F	0.9
T	T	0.99

Bayesian Network Examples

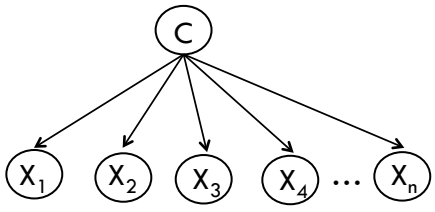


Example: MammoNet



Commonly used Bayesian Networks

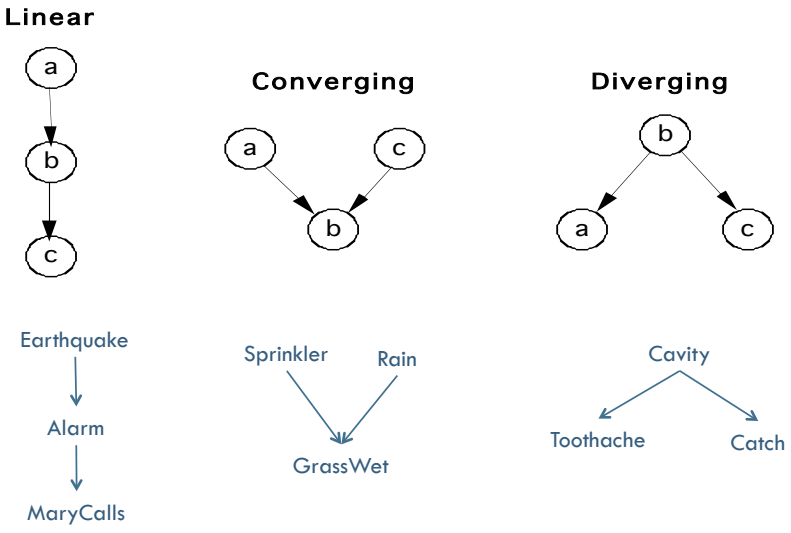
- Naïve (Idiot) Bayes Classifier
  - Commonly used for text classification
  - C is the class (topic or label) of the document
  - The X variables represent the words in the document



Today an earthquake occurred in Southern California…

Independence in BNs

Three Types of Connections



Connection patterns and independence

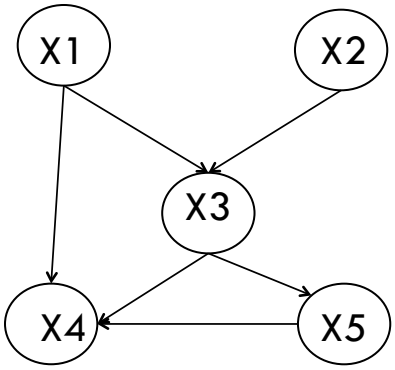
- **Linear connection:** The two end variables are dependent on each other. Observing the middle variable makes them independent.
- **Converging connection:** The two end variables are independent of each other. Observing the middle variable makes them dependent.
- **Divergent connection:** The two end variables are dependent on each other. Observing the middle variable makes them independent.

D-Separation

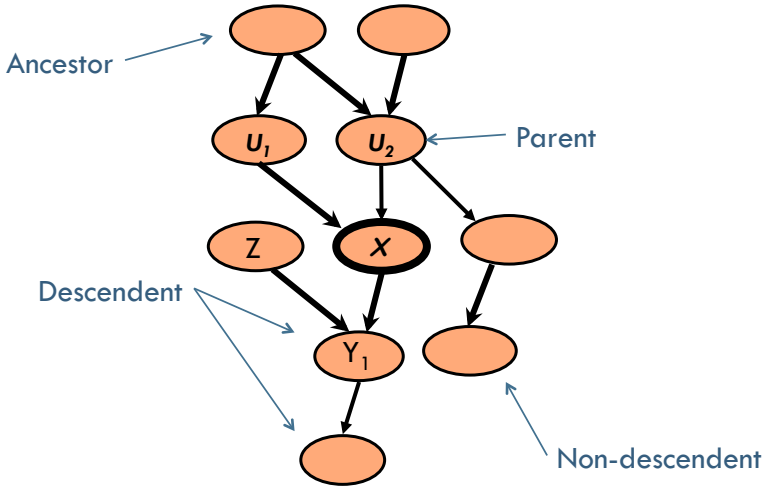
- Algorithm to determine independencies in BN
- Query: Are two variables X_i and X_j independent?
- Check all paths between X_i and X_j
  - If all paths are blocked, then independent
  - If any path is not blocked then not independent

A path is blocked if for any connection on the path the two end variables are independent

List the independencies in the following Bayesian Network

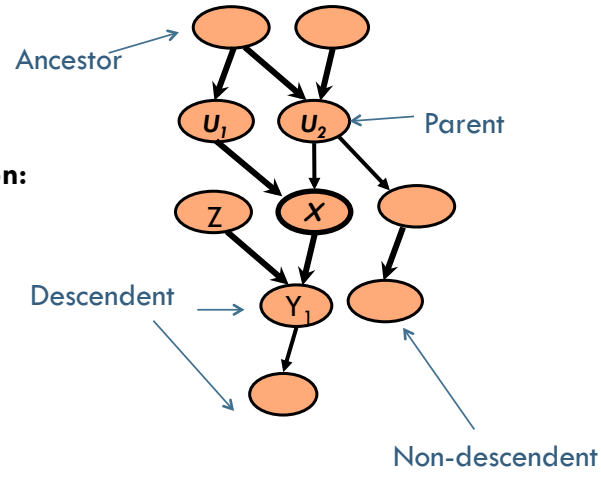


Bayesian Networks terminology



Independence assumptions encoded in the Bayesian Network

Local Markov Assumption:
A node X is independent of its non-descendants given its parents



Independence assumptions encoded in the Bayesian Network

Markov Blanket:
A node X is conditionally independent of all other nodes given its parents, children, and children's parents