# ADVERSARIAL SEARCH

## Today

- Reading
  - AIMA Chapter Read *5.1-5.5*, *Skim 5.7*
- Goals
  - Introduce adversarial games
  - Minimax as an optimal strategy
  - Alpha-beta pruning

# Adversarial Games

- People like games!
- Games are fun, engaging, and hard-to-solve
- Games are amenable to study: precise, easy-to-represent state space

Game pieces found in a burial site in Southeast Turkey. Dated about 3000 BC

"Game of Twenty squares" discovered in a burial site in Ur. Dated about 2550-2400 BC

Backgammon is also among one of the oldest games still played today

# Adversarial Games

- Two-player games have been a focus of AI as long as computers have been around

**Checkers**

Solved: state space was completely mapped out!

**Backgammon and Chess**

Computers can compete at a championship level

**Go**

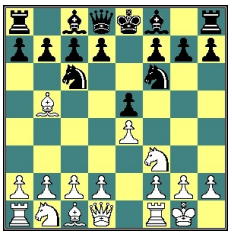
Computers are still at an amateur club-level

# How humans play games

An experiment (by deGroot) was performed in which chess positions were shown to novice and expert players.

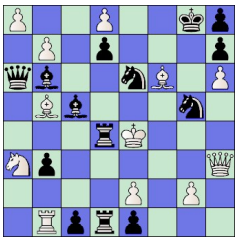
experts could reconstruct these perfectly

novice players did far worse…

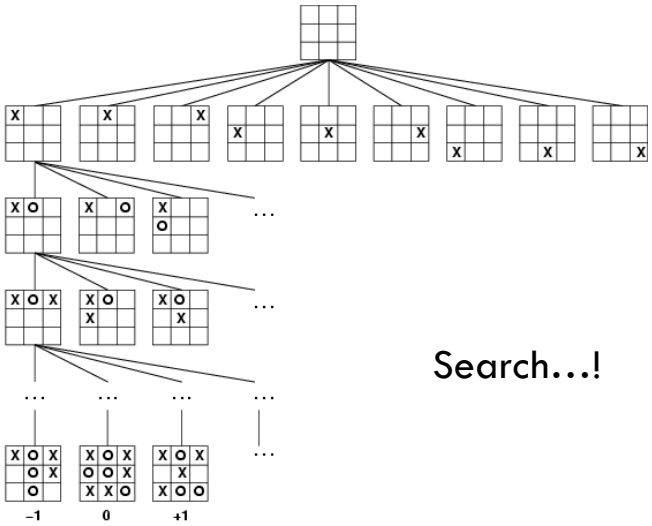


Random chess positions (not legal ones) were then shown to the two groups

experts and novices did just as badly at reconstructing them!



# How computers play games




Search…!

# Terminology

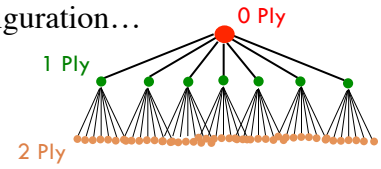
- deterministic vs. stochastic games
- initial state, successor function, goal test,…
- utility function: defines the final numeric value for a game that ends in terminal state s for player p
  - Chess: +1, 0, ½ for a win, loss, or draw
- zero-sum game: equal and opposite utilities
  - If I win, you lose.
  - Chess: 0 +1 , 1 + 0, ½ + ½
- policy: a function that maps from the set of states to the set of possible actions

# Branching factor and depth

On average, there are fewer than 40 possible moves that a chess player can make from any board configuration…

18 Ply!!

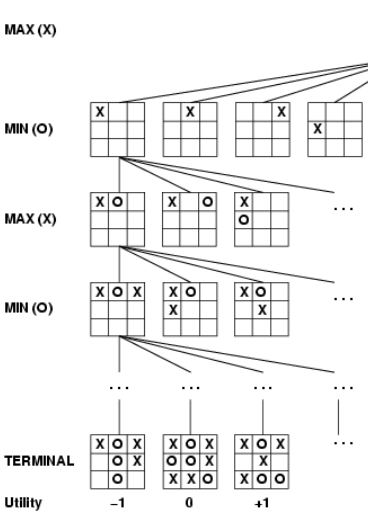
Hydra at home in the United Arab Emirates…




Branching Factor Estimates
for different two-player games

| | |
|---|---|
| Tic-tac-toe | 4 |
| Connect Four | 7 |
| Checkers | 10 |
| Othello | 30 |
| Chess | 40 |
| Go | 300 |

## Simplified representation for two-player games

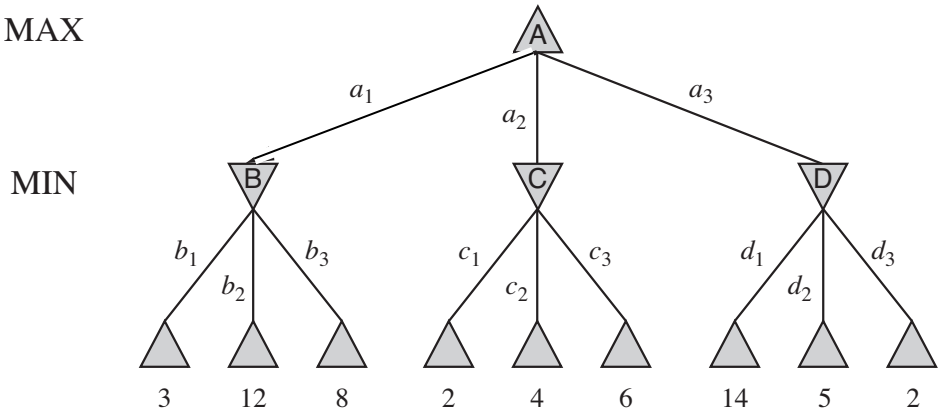



- Two players: MAX and MIN
- MAX always moves first
- MAX wants high utility (payoff)
- MIN wants MAX to get low utility (payoff)

## Minimax: an optimal policy

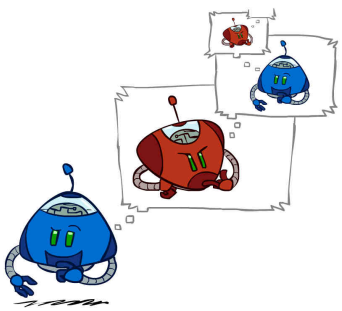
- An optimal policy is one that is at least as good as any other, no matter what the opponent does
  - If there's a way to force the win, it will
  - Will only lose if there's no other option
- Minimax is an optimal policy assuming both players play optimally

# Minimax: an optimal strategy




What action should MAX take?

# Minimax: an optimal policy



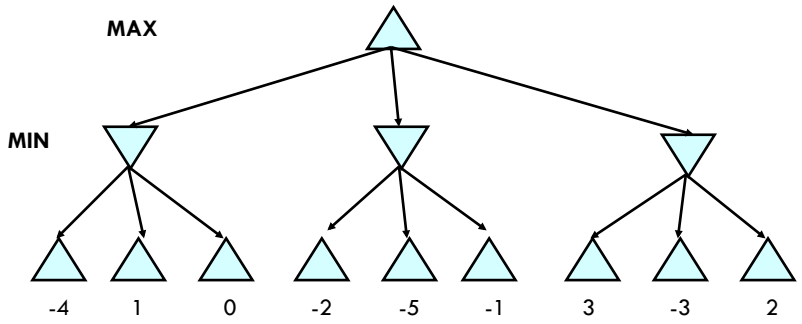
If I did this, then he would do that, but then I would do that, and then he would do this…

$$\text{MINIMAX(s)} = \begin{cases} \text{UTILITY}(s) & \text{if TERMINAL-TEST}(s) \\ \max_a \text{MINIMAX}(\text{RESULT}(s,a)) & \text{if PLAYER}(s) = \text{MAX} \\ \min_a \text{MINIMAX}(\text{RESULT}(s,a)) & \text{if PLAYER}(s) = \text{MIN} \end{cases}$$

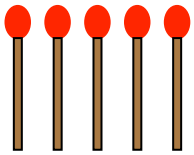
# Minimax: An Optimal Strategy

```
function MINIMAX-DECISION(state) returns an action
    v ← MAX-VALUE(state)
    return the action in SUCCESSORS(state) with value v

function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do
        v ← MAX(v, MIN-VALUE(s))
    return v

function MIN-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← ∞
    for a, s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s))
    return v
```

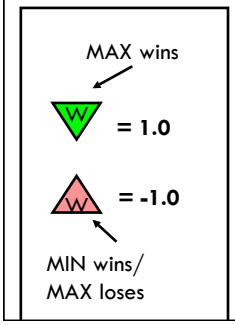
# Minimax Example



# Minimax Example: Baby Nim



Take 1 or 2 at each turn
Goal: take the last match




# Properties of Minimax

- Minimax performs depth-first exploration of game tree.
  - Recall time complexity for DFS is $O(b^m)$
- For chess, b ≈ 35, d ≈100 for "reasonable" games
  - exact solution completely infeasible
- How can we find the exact solution faster?

## Alpha-Beta Pruning

- An extension of minimax
- Prunes the game tree, i.e. eliminates parts of game tree that won't affect the final result
- Alpha-beta returns the same result as minimax but faster

## Alpha-Beta pruning

- alpha is the best scenario MAX has found so far
  - MAX can always achieve a utility of alpha (and hopes for higher)
  - Only MAX modifies alpha
  - MAX uses beta for pruning

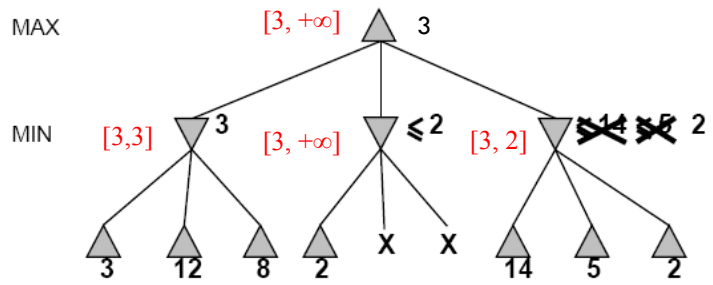
$[\alpha, \beta]$

- beta is the best scenario MIN has found so far
  - MIN can always achieve a utility of beta (and hopes for lower)
  - Only MIN modifies beta
  - MIN uses alpha for pruning

# Alpha-Beta pruning

```
function ALPHA-BETA-SEARCH(state) returns an action
  inputs: state, current state in game

  v ← MAX-VALUE(state, −∞, +∞)
  return the action in SUCCESSORS(state) with value v

function MAX-VALUE(state, α, β) returns a utility value
  inputs: state, current state in game
          α, the value of the best alternative for MAX along the path to state
          β, the value of the best alternative for MIN along the path to state

  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then return v
    α ← MAX(α, v)
  return v

function MIN-VALUE(state, α, β) returns a utility value
  inputs: state, current state in game
          α, the value of the best alternative for MAX along the path to state
          β, the value of the best alternative for MIN along the path to state

  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then return v
    β ← MIN(β, v)
  return v
```

# Alpha-Beta Example



## Properties of $\alpha$ - $\beta$

- Pruning does not affect final result
- However, effectiveness of pruning affected by order in which we examine successors
- What do you do if you don't get to the bottom of the tree on time?