Lecture 8: Induction and Sorting

Today

- Reading
  - JS Ch. 6
- Objectives
  - Selection sort
  - Merge Sort

+ Selection Sort

14	30	10	26	34	18	5	20
5	30	10	26	34	18	14	20
5	10	30	26	34	18	14	20
5	10	14	26	34	18	30	20
5	10	14	18	34	26	30	20

1. Find smallest
2. Swap
3. Repeat

+ Selection Sort

```
/**
 * Sorts an integer array using iterative selection sort
 * @param array array of integers to be sorted
 */
private static void selectionSortIterative(int[] array) {

   for(int i = 0; i < array.length; ++i) {
      int min = indexOfSmallest(array, i);
      swap(array, i, min);
   }
}
```

Selection Sort (helper)

```
/**
 * @param array array of integers
 * @param startIndex valid index into array
 * @return index of smallest value in array[startIndex...n]
 */
protected static int indexOfSmallest(int[] array, int startIndex) {

    int smallest = startIndex;
    for(int i = startIndex+1; i < array.length; ++i) {
        if(array[i] < array[smallest]) {
            smallest = i;
        }
    }
    return smallest;
}
```

Correctness of Selection Sort using Induction (on board)

- Consider what must be true after every iteration of the for-loop in selectionSortIterative

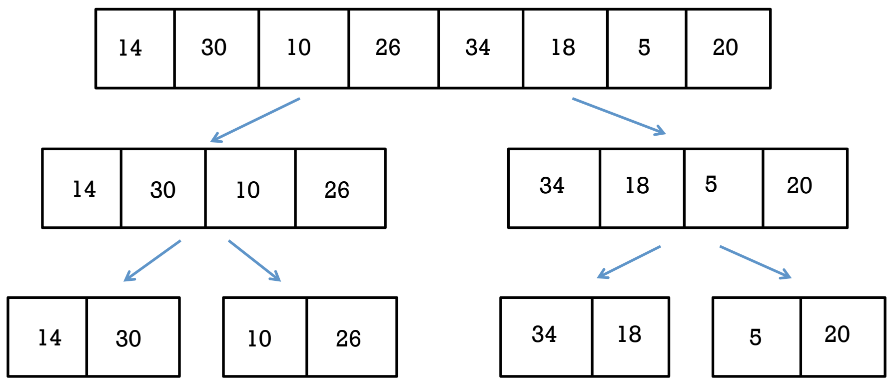
Complexity of Selection sort using Induction (on board)

- Count the number of comparisons performed for each iteration of the for-loop in selectionSortIterative

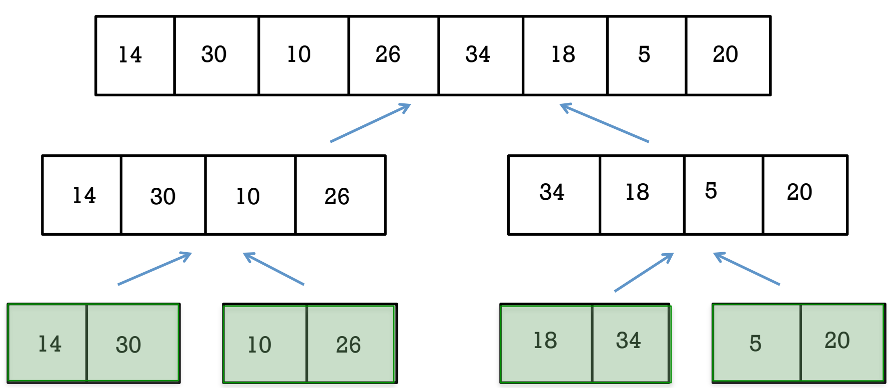
Divide and Conquer

- Divide-and-conquer is a common approach for solving problems
  - Divide the problem into smaller subproblems until the subproblems are so small that the solution is trivial
  - Combine solutions to smaller subproblems to create solution to larger problem
  - Recursion!

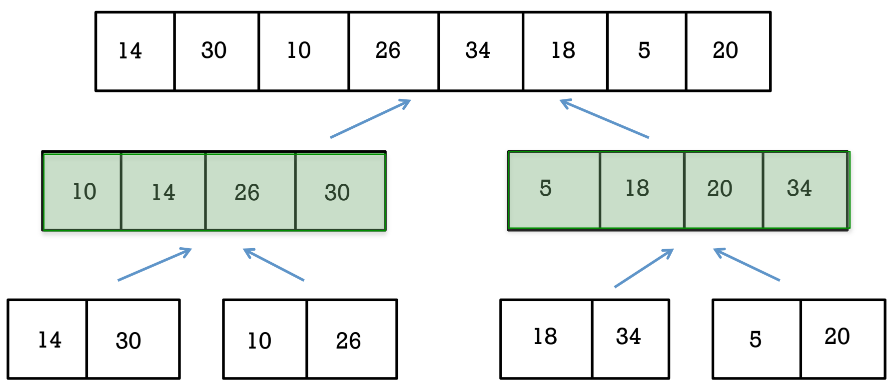
MergeSort



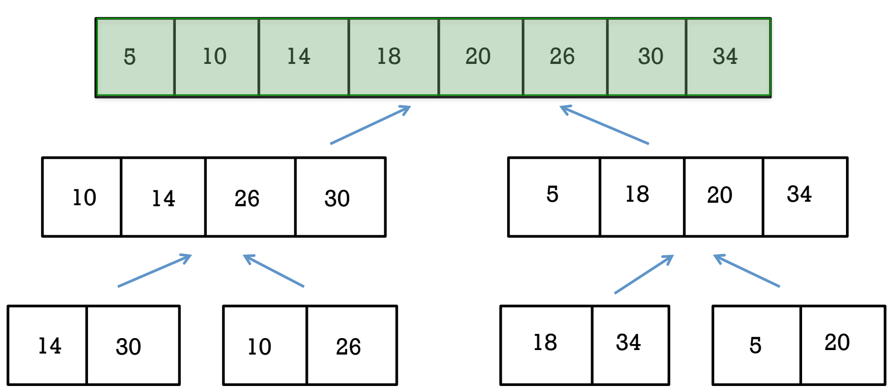
MergeSort



MergeSort



MergeSort



Complexity of Merge Sort

- Let f(n) denote the number of comparisons performed by Merge Sort on an array of size n.
- Then we have the following recurrence relation:

$$f(n) = 2f(n/2) + n$$

- **Claim: $f(n) = n\log_2 n + n$**

Complexity of Merge Sort

- Claim: $f(n) = n\log_2 n + n$
- Proof by Strong Induction
- Base Case
  - Prove for $n = 1$
- Inductive Hypothesis
  - $f(m) = m\log_2 m + m$ **for all** $m < n$
  - Now show that $f(n) = n\log_2 n + n$